23104622

**QP CODE: 23104622**

**Reg No** : .....................

**Name** : .....................

## B.Sc DEGREE (CBCS) REGULAR/IMPROVEMENT/REAPPEARANCE EXAMINATIONS, FEBRUARY 2023

### First Semester

### Complementary Course - MM1CMT01 - MATHEMATICS - PARTIAL DIFFERENTIATION, MATRICES, TRIGONOMETRY AND NUMERICAL METHODS

(Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III , B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 Admission Onwards

C62259A1

Time: 3 Hours Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Describe the level surface of the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$.
2. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y)=\sin^2(x-3y)$.
3. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ if $f(x,y)=x+y+xy$.
4. Define singlar matrix.
5. Write the matrix equation of the system of linear equations 3X+2Y-5Z+4W=4, 4X+3Y+3Z+6W=10, 3X+3Y+2Z+2W=4
6. Define characteristic vector of a square matrix.
7. Express $\sin 3\theta$ in terms of $\sin\theta$.
8. Prove that $\sinh(x-y)=\sinh x\cosh y-\cosh x\sinh y$.
9. Separate $\cosh\left(\alpha-i\beta\right)$ into its real and imaginary parts.

10. Write the binomial expansion of $(1+x)^n$, when $n$ is a positive integer and when $n$ is a rational number.

11. Use the method of false position to compute the first approximation to a root of the equation $x^3 - x - 1 = 0$, given that the root lies between $1$ and $2$.

12. Give the generalized Newton's formula to find a root of $f(x) = 0$ with multiplicity $p$.

(10×2=20)

**Part B**

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Verify whether the function $f(x,y) = \ln\sqrt{x^2+y^2}$ satisfies the two-dimensional Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

14. Use chain rule to evaluate $\frac{dw}{dt}$ at $t = 1$ if
$w = z - \sin(xy),\ x = t,\ y = \ln t,\ z = e^{t-1}$.

15. find $\frac{\partial w}{\partial v}$ when $u = 0, v = 0$ if $w = x^2 + \left(\frac{y}{x}\right),\ x = u - 2v + 1,\ y = 2u + v - 2$.

16. Obtain the column equivalent canonical matrix $A$ to the following matrix and hence find its rank $A = \begin{bmatrix} 3 & 1 & 2 & 5 \\ -1 & 4 & 1 & -1 \\ 1 & 9 & 4 & 3 \end{bmatrix}$

17. If $A$ and $B$ are two square matrices such that $B$ is non-singular, then show that $AB$ and $BA$ have the same characteristic roots.

18. If $x$ is real, show that $\sinh^{-1}(x) = \log(x + \sqrt{x^2+1})$.

19. Sum to infinity the series $1 + \frac{c\cos\theta}{1!} + \frac{c^2\cos 2\theta}{2!} + \frac{c^3\cos 3\theta}{3!} + \ldots$

20. Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method.

21. Apply Newton - Raphson method to solve the algebraic equation $f(x) = x^3 + x - 1 = 0$ correct to $6$ decimal places. (Start with $x_0 = 1$)

(6×5=30)

**Part C**

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Solve the system of equations $2x - y + 3z = 0, 3x + 2y + z = 0, x - 4y + 5z = 0$

23.

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and then verify Cayley-Hamilton theorem.
2. Express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in $A$.

24. (a) Expand $\cos^3\theta \sin^4\theta$ in a series of cosines of multiples of $\theta$.

(b) Sum to infinity the series $c\cos\alpha + \frac{c^2}{2}\cos 2\alpha + \frac{c^3}{3}\cos 3\alpha + \ldots$

25. State and prove the theorem, which gives a sufficient condition for convergence of the iteration process in the iteration method
for finding the roots of a given equation.

(2×15=30)