

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012**Fourth Semester****DIFFERENTIAL EQUATIONS, ABSTRACT ALGEBRA, NUMERICAL ANALYSIS**

(Complementary Course to Physics / Chemistry / Petrochemicals / Geology,

Food Science and Quality Control and Computer Maintenance and Electronics)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type Questions)*Answer all questions.**Each bunch of 4 objective questions has weight 1.*

- I. 1 Find a solution of the differential equation $\frac{dy}{dx} = \cos 5x$.
- 2 Give an example of an exact differential equation.
- 3 Find an integrating factor of the differential equation $(2y^2 + 3x) dx + 2xy dy = 0$.
- 4 What is the order of the differential equation :

$$\left(\frac{dy}{dx} - 1\right)\left(y - x \frac{dy}{dx}\right) = \frac{dy}{dx} ?$$

- II. 5 Write down the equation of a three dimensional surface.
- 6 Give an example of first order partial differential equation in three variables.
- 7 What is the general form of a first order partial differential equation ?
- 8 Form a partial differential equation from the surface $x^2 + y^2 = (z - c)^2 = a^2$.
- III. 9 Write down the formula for calculating the absolute error.
- 10 Give an example of a transcended function.
- 11 In the method of false position to find a root of $f(x) = 0$ between x_0 and x_1 , what we are replacing by the part of the curve between $(x_0, f(x_0))$ and $(x_1, f(x_1))$?
- 12 What is the Newton-Raphson formula ?
- IV. 13 Give an example of a cyclic group of order 8.
- 14 What is the order of the subgroup of \mathbb{Z}_{12} generated by 3 ?

Turn over

- 15 How many elements are there in the group of symmetries of an equilateral triangle ?
 16 Give an example of a group homomorphism from \mathbb{Z} to $5\mathbb{Z}$.

(4 × 1 = 4)

Part B (Short Answer Questions)

*Answer any five questions.
 Each question has weight 1.*

- 17 Find the general solution of the differential equation $x \frac{dy}{dx} + (3x+1)y = e^{-3x}$
 18 Solve $\left(\frac{dy}{dx}\right)^2 = y - x$.
 19 Eliminate the constants a and b from the equation $ax^2 + by^2 + z^2 = 1$.
 20 Find $\frac{\partial(F, G)}{\partial(x, y)}$ if $F = ax^2 + by^2 + cz^2 - 1$ and $G = x + y + z - 1$.
 21 Find an approximate root of $x^3 - x - 4 = 0$ using bisection method.
 22 Find an approximate root of $x = \frac{1}{(1+x)^2}$ using the iterative method.
 23 Draw the lattice diagram for subgroup of S_3 .
 24 Check whether the set of integers \mathbb{Z} over the real numbers \mathbb{R} is a vector space with usual addition and multiplication.

(5 × 1 = 5)

Part C (Short Essay Questions)

*Answer any four questions.
 Each question has weight 2.*

- 25 Verify whether $(x + \sin y)dx + (y^2 + x \cos y)dy = 0$ is exact. If exact, solve it.
 26 Solve $xy^2 \frac{dy}{dx} + y^3 = 1$
 27 Find the general integral linear partial differential equation : $z(xp - yq) = y^2 - x^2$.

- 28 Find the integral curves of the equations $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.
- 29 Use Newton-Raphson method to obtain a root to three decimal places of the equation $x^3 - 2x - 5 = 0$.
- 30 Show that the set of all $m \times n$ matrices with entries from \mathbb{R} under usual addition is an abelian group.

(4 × 2 = 8)

Part D

*Answer any two questions.
Each question has weight 4.*

- 31 Solve $\frac{dy}{dx} - y = e^x y^2$.
- 32 Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ using the method of false position.
- 33 Show that the set of all rational numbers with usual addition and multiplication is a field.

(2 × 4 = 8)