

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014**Fourth Semester****FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND
ABSTRACT ALGEBRA**

(2011 Admission onwards)

[Complementary Course to Physics, Chemistry, Petrochemicals, Geology, Food Science and
Quality Control and Computer Maintenance and Electronics]

Time : Three Hours

Maximum Weight : 25

Part A*Answer all questions.**Each bunch of four questions has weight 1.*

- I. 1 Find the smallest positive period of $\sin 2\pi x$.
 2 Determine whether the function $x|x|$ even, odd or neither even nor odd.
 3 Write the Legendre's equation.
 4 Write Bessel function of the first kind of order n .
- II. 5 Form a partial differential equation by eliminating the arbitrary constants from the equation $z = ax + by + a$.
 6 Write a parametric equation of the spherical surface $x^2 + y^2 + z^2 = a^2$.
 7 Write the Lagrange's partial differential equation.
- 8 If $F = ax^2 + by^2 + cz^2 - 1$ and $G = x + y + z - 1$, find $\frac{\partial(F, G)}{\partial(y, z)}$.
- III. 9 If $X = 0.51$ and is correct to 2 decimal places, find ΔX .
 10 Round off 38.46235 to four significant figures.
 11 Find the relative error of the number 8.6 if both of its digits are correct.
 12 Write the Maclaurin expansion of $\sin x$.
- IV. 13 Let $*$ be defined on \mathbb{Q} , the set of rationals by letting $a * b = ab$. Is $(\mathbb{Q}, *)$ a group.
 14 Find the order of the cyclic subgroup of \mathbb{Z}_4 generated by 3.
 15 Define a symmetric group of n letters.
 16 Give a basis for the vector space $\mathbb{R}(\sqrt{2})$ over \mathbb{R} .

(4 × 1 = 4)

Turn over

Part B

Answer any **five** questions.
Each question has weight 1.

17. Sketch the graph of $f(x) = x^2$, for $-\pi < x < \pi$.
18. Write the first four Legendre polynomials.
19. Derive a partial differential equation by eliminating f and ϕ from the relation

$$z = f(x + ay) + \phi(x - ay).$$
20. Find the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$,
 $x + y + z = 1$.
21. Find the quotient $q = x/y$, where $x = 4.536$ and $y = 1.32$ both x and y being correct to the digits given.
22. Write the technique involved in the false position method.
23. Find all orders of the subgroups of Z_6 .
24. Let V be a vector space over a field F . Define a subspace of V over F . Also show that intersection of subspaces of V is again a subspace of V over F .

(5 × 1 = 5)

Part C

Answer any **four** questions.
Each question has weight 2.

25. Find the Fourier series for the function
$$f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ \pi - x & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$
26. Define Gamma function. Show that $\Gamma(n+1) = n!$ for $n = 0, 1, 2, \dots$
27. Find the integral curves of $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.
28. Find a real root of $x^3 - x - 1 = 0$ correct to three decimal places by false position method.
29. Use bisection method to obtain a root correct to three decimal places for $x^3 - 18 = 0$.
30. Prove that a subgroup of a cyclic group is cyclic.

(4 × 2 = 8)

Part D

*Answer any two questions.
Each question has weight 4.*

31. Find the Fourier cosine series as well as the sine series for the function $f(x) = \pi - x$, $0 < x < \pi$.
32. Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$.
33. Using iteration method to find a positive root between 0 and 1 of the equation $xe^x = 1$.

(2 × 4 = 8)