

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2013****Fourth Semester****FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND  
ABSTRACT ALGEBRA**

(Complementary Course to Physics, Chemistry, Petrochemicals, Geology, Food Science and  
Quality Control and Computer Maintenance and Electronics)

[2011 Admission onwards]

Time : Three Hours

Maximum Weight : 25

**Part A**

*Answer all questions.*

*Each bunch of four questions has weight 1.*

- I. 1 What is fundamental period of a function  $f(x)$  ?  
2 Write the Euler formula for Fourier coefficients.  
3 Define a power series.  
4 What is Legendre's polynomial of degree  $n$ .
- II. 5 Write the parametric equations of a surface.  
6 What are direction cosines ?  
7 Write the partial differential equation representing the set of all spheres with centers on the  $z$ -axis.  
8 Consider the equation  $x \frac{dz}{dx} = z^2 + x^2$ . Is this equation linear ?
- III. 9 Round off the number 81.255 to two decimal places.  
10 Define the term absolute error.  
11 Find the Maclaurin series expansion of  $e^x$ .  
12 Write Newton-Raphson formula for approximation.
- IV. 13 Let  $*$  be defined on  $2\mathbb{Z} - \{2n \mid n \in \mathbb{Z}\}$  by letting  $a * b = a - b$ . Determine whether the binary operation  $*$  gives a group structure on  $2\mathbb{Z}$ .  
14 Write a non-trivial proper subgroup of  $\mathbb{Z}_4$ .

Turn over

15 If  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$   $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$  Find  $\sigma\tau$

16 Define a ring homomorphism.

(4 × 1 = 4)

### Part B

*Answer any five questions.  
Each question has weight 1.*

17 Sketch the graph of  $f(x) = |x|$  in the interval  $-\pi < x < \pi$ .

18 Write the formula for the Bessel functions  $J_0(x)$  and  $J_1(x)$ .

19 Find the condition that the plane  $lx + my + nz + p = 0$  should touch the central Conicoid  
 $ax^2 + by^2 + cz^2 = 1$ .

20 Eliminate the arbitrary function  $f$  from the equation  $z = xy + f(x^2 + y^2)$ .

21 Calculate the value of  $\sqrt{102} - \sqrt{101}$  correct to four significant figures.

22 Obtain the range of values within which the exact value of  $\frac{1.265(10.21 - 7.54)}{47}$  lies, if all the numerical quantities are rounded-off.

23 Describe all the elements in the cyclic subgroup of  $GL(2, \mathbb{R})$  generated by the matrix  $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

24 Determine whether the given set of vectors is a basis for  $\mathbb{R}^3$  over  $\mathbb{R}$ .

$$[(-1, 1, 2), (2, -3, 1), (10, -14, 0)]$$

(5 × 1 = 5)

### Part C

*Answer any four questions.  
Each question has weight 2.*

25 Find the Fourier series for the function  $f(x) = \begin{cases} -2x & \text{if } -\pi < x < 0 \\ 2x & \text{if } 0 < x < \pi \end{cases}$ .

26 Solve the equation  $y' - y = 0$  using power series method.



- 27 Find the integral curves of the equations  $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$ .
- 28 Use bisection method to obtain a root correct to 3 decimal places for the equation  $x^3 - x^2 - 1 = 0$ .
- 29 Use newton-Raphson method to obtain a root of  $x^3 + 3x^2 - 3 = 0$  correct to 3 decimal places.
- 30 If  $G$  is a group with binary operation  $*$  and if  $a$  and  $b$  are any elements of  $G$ , then show that the linear equation  $a * x = b$  and  $y * a = b$  have unique solutions  $x$  and  $y$  in  $G$ .

(4 × 2 = 8)

**Part D**

*Answer any two questions.  
Each question has weight 4.*

- 31 Find the Fourier series of the periodic function  $f(x) = 1 - x^2$ ,  $-1 < x < 1$  and sketch  $f(x)$ .
- 32 Find the general integrals of the linear partial differential equation  $y^2 p - xyq = x(z - 2y)$ .
- 33 Use quotient-difference method to obtain the approximate roots of the equation  $x^3 - x^2 - 2x + 1 = 0$ .

(2 × 4 = 8)