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Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2017

Second Semester

Complementary Course—Mathematics

INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\int \sin^2 x \, dx$.
2. Evaluate $\int x \cos x \, dx$.
3. State Fundamental theorem of Calculus.
4. Write the formula for the volume of a solid of revolution about y-axis.
5. Define a smooth function.
6. Write the formula for calculating the area of a closed bounded region in polar co-ordinates.
7. Write the formula for calculating the length of a smooth curve $x = g(y)\; c \le y \le d$.
8. Define rank of a matrix.
9. What is the characteristic equation of a matrix A.
10. State Cayley-Hamilton theorem.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Evaluate $\int \frac{2z\,dz}{\sqrt[3]{z^2+1}}$.

Turn over

12. Evaluate $\int_{-\pi/3}^{\pi/3} \frac{1-\cos 2t}{2}\, dt$.

13. Suppose $\int_1^3 f(x)\, dx = 5$. Find $\int_1^2 \sqrt{3}\, f(z)\, dz$.

14. Find the area of the region enclosed by $x = y^2$ and $x = y + 2$.

15. Find the volume of the solid generated by revolving $y = x^3, y = 0, x = 2$ about the x-axis.

16. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

17. Evaluate $\int_0^1 \int_2^{4-2x} dy dx$.

18. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x$.

19. Evaluate $\int_0^1 \int_0^2 \int_0^{1-y} dz\, dx\, dy$.

20. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find A^2 using Cayley-Hamilton theorem.

21. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

22. What are the elementary transformations of a matrix.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Express the solution of the initial value problem $\frac{dy}{dx} = \sec x, y(2) = 3$ interms of integrals.

24. Find the area of the region between the x-axis and the graph of $y = x^3 - x^2 - 2x, -1 \le x \le 2$.

25. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

26. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = 3/2$ to generate a solid. Find the volume of the solid.

27. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx$ and write an equivalent integral with the order of integration reversed.

28. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \le x \le \pi,\ 0 \le y \le \pi/2$.

29. Evaluate using polar integrals $\int_0^1 \int_0^{\sqrt{1-y^2}} \left(x^2 + y^2\right) dx\, dy$.

30. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$.

31. Show that if λ is a characteristic root of a non-singular matrix A, then λ^{-1} is a characteristic root of A^{-1}.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Evaluate $\iint_R e^{x^2 + y^2}\, dy dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$.

(b) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

33. (a) Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$.

(b) Evaluate $\int_0^2 \int_x^2 2y^2 \sin xy \, dy \, dx$.

34. (a) Find that area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$ about the x-axis.

(b) Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_a^b f(x)\, dx = 0$, then $f(x) = 0$ atleast once in $[a, b]$.

35. (a) Solve the system of equations :

$$x + y + z + w = 0$$

$$x + y + z - w = 4$$

$$x + y - z + w = -4$$

$$x - y + z + w = 2.$$

(b) If $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. Find its eigen value and the corresponding eigen vectors.

(2 × 15 = 30)