E 6122 (Pages : 3) Reg. No.................................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Fourth Semester

Complementary Course—Mathematics

FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

[For the programme B.Sc. Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics]

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Check whether $f(x) = |x^3|$ is an odd function.
2. Write the Bessel function of the second kind of order v.
3. Write the partial differential equation representing the family of concentric circles.
4. Write the Newton-Raphson formula.
5. Write the Rodrigue's formula.
6. Find the cube root of 2 correct to three decimals.
7. If $y = 4x^3 - 0.16x$. Find the percentage error in y at $x = 0.5$, if the error in x is 0.35.
8. Find the order of the element 2 in the group $\mathbb{Z}_4$.
9. Define homomorphism. How many homomorphisms are there from $\mathbb{Z}_2$ onto $\mathbb{Z}_2$.
10. Give a basis for $Q\left(\sqrt{3}\right)$ over Q.

$(10 \times 1 = 10)$

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find the Fourier, series for the function $f(x) = 3x\,(\pi^2 - x^2)\ (-\pi < x < \pi)$.

12. Eliminate the function from $z = f\left(\frac{xy}{z}\right)$.

13. Form a partial differential equation by eliminating a and b from $2z = (ax + y)^2 + b$.

14. Find the integral curves of the equation $\frac{dy}{cy - bz} = \frac{dy}{az - cx} = \frac{dz}{bx - ay}$, show that they are circles.

15. Evaluate $\sqrt{3} + \sqrt{11}$ correct to 4 decimal places.

16. Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to three significant digits.

17. Find a real root of the equation $f(x) - x^3 - 3x^2 + 5x - 10$ using bisection method.

18. Find the $\sqrt[3]{23}$ correct to 4 decimal places by Newton's method.

19. Prove that order of an element in a finite group divides the order of the group.

20. Find all the subgroups of $\mathbb{Z}_{12}$.

21. Find the order of each element in $\mathbb{Z}_2 \oplus \mathbb{Z}_4$.

22. Prove that every subgroup of a cyclic group is cyclic.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. A function $f(x)$ is defined with in the range $(0, 2\pi)$ by $f(x) = \begin{cases} x & , \quad 0 < x < \pi \\ 2\pi - x & , \quad \pi < x < 2\pi \end{cases}$ Express $f(x)$ as a Fourier series in $(0, 2\pi)$.

24. Solve $x^2p + y^2q = nxy$.

25. Show that $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$.

26. Show that $nP_n(x) = xP'_n - P'_{n-1}$

27. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

28. Using Newton-Raphson method, find correct to four decimals the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.

29. Expand $f(x) = e^x \sin x$ as a Maclaurin series up to the term containing x^4 and compute the value of $\sin 34^\circ$.

30. Prove that an integer k in $\mathbb{Z}_n$ is a generator of $\mathbb{Z}_n$ if and only if $gcd(k, n) = 1$.

31. Define $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_{12}$ by $\phi(x) = 3x$. Find the kernel of ϕ.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 marks.

32. (a) Find the Fourier sine series and cosine series of of $f(x) = x^3$ $(0 < x < L)$.

(b) Find the Fourier series of $f(x) = \frac{x^2}{2}$ $(-\pi < x < \pi)$. Deduce, $1 - \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots\ldots = \frac{\pi^2}{2}$.

33. (a) Solve $(y^2 + z^2) p - xyq = -zx$.

(b) Find the integral surface of the $(x - y)y^2 p + (y - x) x^2 q = (x^2 + y^2) z$ through the curve $xz = a^3$, $y = 0$.

34. (a) Using Newton-Raphson method, find correct to four decimals the root of the equation $\sin x = 1 + x^3$ llies between -2 and -1.

(b) Find the root of the equation $x^3 - 2x - 5 = 0$ by Regula-Falsi method, when it is given that the root lies between 2 and 3.

35. (a) Show that the set of all complex numbers with usual addition and multiplication forms a field. Is it a vector space over $\mathbb{C}$? Justify.

(b) Obtain the multiplication table for the cyclic subgroup S_5 generated by the permutation $\sigma = (1\ 2\ 4)\ (3\ 5)$. Is this group isomorphic to S_3.

(2 × 15 = 30)