E 4260 (Pages : 3) Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012

Third Semester

Complementary Course—VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

[Common for (1) Physics, (2) Chemistry, (3) Petrochemicals, (4) Geology, (5) Computer Maintenance and Electronics, (6) Food Science and Quality Control]

[For 2011 Admission onwards]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions carries weight* 1.

I. 1 Define continuity of a vector function $r(t)$ at a point.

2 Find the length of one turn of the helix $r(t) = \cos t\, i + \sin t\, j + tk$.

3 What is the curvature of a straight line ?

4 Define gradient of $f(x, y)$ at a point.

II. 5 Write the formula for calculating flux across a smooth closed plane curve.

6 Whether the field $\mathrm{F} = yz\bar{i} + xz\bar{j} + xy\bar{k}$ conservative.

7 Find the divergence of :

$$\mathrm{F}(x, y) = (x^2 - y^2)i + (xy - y)j$$

8 State Stoke's theorem.

III. 9 Is the equation $(e^y + 1)\cos x\, di + e^y \sin x\, dy = 0$ exact.

10 Which type of equations are called Clairaut's equations.

11 Write the general form of Bernoulli's equation.

12 Write the integrating factor of $(y - x^2)\,dx + (x^2 \sin y - x)\,dy = 0$.

IV. 13 Find the directrix of the parabola $x = 2y^2$.

14 Write the equation of the hyperbola in standard form where $\mathrm{Foci}\left(0, \pm\sqrt{2}\right)$ and asymptotes $\left(0, \pm\sqrt{5}\right)$.

15 Find the eccentricity of the hyperbola $y^2 - 3x^2 = 3$.

16 Which curve is represented by the equation $x^2 + 2xy + y^2 + 2x - y + 2 = 0$.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Find the point on the curve $r(t) = 12 \sin t\, i - 12 \cos t\, j + 5t\, \bar{k}$ at a distance 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.

18 What is a differentiable curves unit tangent vector ? Give *one* example.

19 Evaluate $\int_c (x+y)\,dS$, where c is the straight line segment $x = t, y = 1-t, z = 0$ from (0, 1, 0) to (1, 0, 0).

20 Find the work done by the force $F = xyi + yj - yz\bar{k}$ over the curve $r(t) = t\,i + t^2 j - t\,\bar{k}$, $0 \leq t \leq 1$.

21 Solve the equation $p^2 - 5p + 6 = 0$.

22 Solve the equation :

$$y - x\frac{dy}{dx} = x + y\frac{dy}{dx}.$$

23 The ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ is shifted 3 units to the left and 2 units down to generate the ellipse $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{25} = 1$. Find the foci, vertices and center of the new ellipse.

24 The x and y axes are rotated through an angle $\pi/4$ radians about the origin. Find an equation for the hyperbola $2xy = 9$ in the new co-ordinates.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Find the curvature of the helix $r(t) = (a \cos t)\, i + (a \sin t\, j) + bt\, k\; a, b \geq 0, a^2 + b^2 \neq 0$.

26 Verify Green's theorem for $F(x, y) = (x - y)\, i + xj$ and the region $\mathbb{R}$ bonded by the unit circle $r(t) = (\cos t)\, i + (\sin t) j + 0 \leq t \leq 2\pi$.

27 Verify Gauss's divergence theorem for the field $F = xi + yj + z\bar{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

28 Solve the equation $\frac{dy}{dx} + y \cos x = y^n \sin 2x$.

29 Sketch the hyperbola $8y^2 - 2x^2 = 16$.

30 Find a polar equation for an ellipse with semi major axis 39.44 AU and eccentricity 0.25.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Use surface integral in Stoke's theorem to calculate the circulation of the field $F = x^2 i + 2xj + z^2 k$ around the curve c, the ellipse $4x^2 + y^2 = 4$ in the xy plane, counter clockwise.

32. (a) Solve the equation :

$$xyp^2 - (x^2 + y^2)p + xy = 0$$

(b) Solve the equation :

$$y^2 - (y - xp) = x^4 p^2$$

33. (a) Sketch the circle $r = 4\cos\theta$. Give polar co-ordinates for the centers and identify the radius.

(b) A wheel of radius a rolls along a horizontal straight line. Find parametric equation for the path traced by a point P on the wheel's circumference.

$(2 \times 4 = 8)$