

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2016**Second Semester**

Complementary Course—Mathematics

INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A

*Answer all questions.
Each question carries 1 mark.*

1. Evaluate $\int \cos \frac{x}{2} dx$.
2. State mean value theorem for definite integrals.
3. Evaluate $\int_0^{\pi} (1 + \cos x) dx$.
4. Write the formula for calculating the volume of a solid of revolution about the x -axis.
5. Write the surface area formula for revolution about the x -axis.
6. Find the area of the region bounded by $y = x$ and $y = x^2$ in the first quadrant.
7. How to change Cartesian integrals into polar integrals ?
8. State Cayley-Hamilton theorem.
9. What is the rank of a non-singular matrix of order n .
10. What are the elementary row transformations ?

(10 × 1 = 10)

Part B

*Answer any eight questions.
Each question carries 2 marks.*

11. Evaluate $\int \frac{1}{t^2} \cos \left(\frac{1}{t} - 1 \right) dt$.

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12. Evaluate $\int_{-4}^4 |x| dx$.
13. Suppose f is continuous and that $\int_0^3 f(z) dz = 3$ and $\int_0^4 f(z) dz = 7$. Find $\int_4^3 f(t) dt$.
14. Find the area between the curves $y = x^2$ and $y = -x^2 + 4x$.
15. Find the volume of the solid generated by revolving $y = x - x^2, y = 0$ about x -axis.
16. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, 0 \leq x \leq 1$.
17. Evaluate $\int_0^1 \int_y^{\sqrt{y}} dx dy$.
18. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.
19. Evaluate $\int_0^2 \int_0^1 \int_0^{1-x} dy dz dx$.
20. By reducing to the normal form, find the rank of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}$.
21. If A is an $n \times n$ non-singular matrix. Show that $(A^T)^{-1} = (A^{-1})^T$.
22. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find A^{-1} .

(8 × 2 = 16)

Part C

Answer any **six** questions.
Each question carries 4 marks.

23. Express the solution of the initial value problem $\frac{dy}{dx} = \sqrt{1+x^2}$, $y(1) = -2$ in terms of integrals.
24. Find $\frac{dy}{dx}$ where $y = \int_0^{\sqrt{x}} \sin(t^2) dt$.
25. Find the area of the region in the 1st quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$, by integrating with respect to y .
26. A pyramid 3 m. high has a square base that is 3 m. on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.
27. Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy -plane bounded by the x -axis, the line $y = x$ and the line $x = 1$.
28. Sketch the region bounded by lines $x = 0$, $y = 2x$ and $y = 4$. Express the region's area as an iterated double integral and evaluate the integral.
29. Evaluate using polar integrals $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy dx$.
30. Prove that the rank of the transpose of a matrix is same as that of the original matrix.
31. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 2 & -3 & -4 & 6 \\ 1 & 3 & 1 & -2 & 4 \\ 2 & 5 & -2 & -5 & 10 \end{bmatrix}$.

(6 × 4 = 24)

Turn over

Part D

Answer any **two** questions.
Each question carries 15 marks.

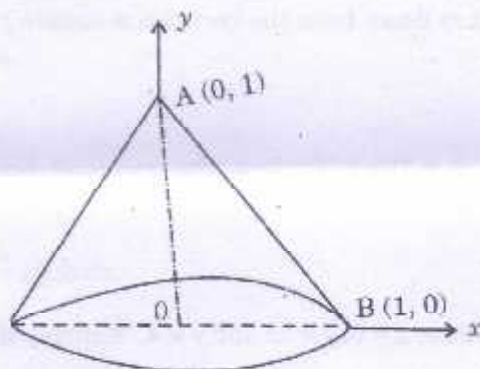
32. (a) Find the area of the region common to the interiors of the cardioids $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$.

- (b) Find the volume of the region between the cylinder $z = y^2$ and the xy -plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

33. (a) Evaluate $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$.

- (b) The line segment $x = 1 - y, 0 \leq y \leq 1$ is revolved about the y -axis to generate the cone as in the figure below :

Find its lateral surface area.



34. (a) Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

- (b) Show that the value of $\int_0^1 \sqrt{1 + \cos x} dx$ cannot possibly be 2.

35. (a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.

- (b) Solve the system of equations :

$$5x + 3y + 3z = 48$$

$$2x + 6y - 3z = 18$$

$$8x - 3y + 2z = 21.$$

(2 × 15 = 30)