Reg. No...............................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2014

Third Semester

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Computer Maintenance and Electronics and Food Science and Quality Control)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Write the vector function representing a Helix.
2. Show that $u(t) = \sin(t)\, i + (\cos t)\, j + \sqrt{3}\, k$ is orthogonal to its derivative.
3. Find the unit tangent vector to the curve $r(t) = (2\cos t)\, i + (2\sin t)\, j + \sqrt{5}\, t\, k, 0 \le t \le \pi$.
4. Define gradient field of a differentiable function $f(x, y, z)$.
5. Write the formula for calculating the flux across a smooth closed plane curve.
6. State Green's theorem.
7. State Stoke's theorem.
8. Define an exact differential equation.
9. Write the asymptotes of $\frac{x^2}{4} - \frac{y^2}{5} = 1$.
10. Find the eccentricity of the ellipse $3x^2 + 2y^2 = 6$.

$(10 \times 1 = 10)$

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find N for the curve $r(t) = (\cos t + t \sin t)\, i + (\sin t - t \cos t)\, j, t > 0$.

12. Find τ for the curve
    $r(t) = (3 \sin t)\, i + (3 \cos t)\, j + 4t\, k$.

13. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $A = 3i - 4j$.

14. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from (1, 2, 3) to (0, –1, 1).

15. Find the work done by F over the curve in the direction of increasing t, where $F = 2yi + 3xj + (x + y)k,\ r(t) = (\cos t)\, i + (\sin t)\, j + (t/6)\, k, 0 \le t \le 2\pi$.

16. Evaluate $\oint_C xydy - y^2\, dx$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

17. Solve the equation
    $\left(2x + e^y\right) dx + xe^y\, dy = 0$.

18. Solve $y = px + \log p$, where $p = \dfrac{dy}{dx}$.

19. Solve $y = p \sin p + \cos p$.

20. Find the Cartesian co-ordinates of $\left(\sqrt{2}, \pi/4\right)$.

21. Find the polar equation of the hyperbola with eccentricity 3/2 and directrix $x = 2$.

22. Describe the motion of a particle whose position $p(x, y)$ at time t is given by $x = a \cos t$, $y = b \sin t, 0 \le t \le 2\pi$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. The velocity of a particle moving in space is $\frac{dv}{dt} = (\cos t)\, i - (\sin t)\, j + k$. Find the particles position as a function of t if $r = 2i + k$, when $t = 0$.

24. Find the curvature for the curve $r(t) = \left(e^t \cos t\right) i + \left(e^t \sin t\right) j + 2k$.

25. Write an equation for the tangent line to the curve $x^2 - y = 1$ at $\left(\sqrt{2}, 1\right)$.

26. Find a potential function f for the field $\mathrm{F} = (y+z)\, i + (x+z)\, j + (x+y)\, k$.

27. Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$.

28. Use divergence theorem to find the outward flux of F across the boundary of the region D, where $\mathrm{F} = (y-x)\, i + (z-y)\, j + (y-x)\, k$ and D is the cube bounded by the planes $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$.

29. Solve the equation

    $ydx - xdy + \left(x^2 + y^2\right) dx = 0.$

30. Solve the equation $\frac{dy}{dx} - y \tan x = e^x \sec x$.

31. Sketch the conic $16x^2 + 25y^2 = 400$.

    Include the foci in your sketch.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 marks.

32. (a) Write a in the form $a_{\mathrm{T}}\mathrm{T} + a_{\mathrm{N}}\mathrm{N}$ at the given value of t without finding T and N, where $r(t) = (t+1)\, i + 2t\, j + t^2\, k, t = 1$.

(b) Find the directions in which $f(x, y) = x^2 + xy + y^2$ increase and decrease most rapidly at $(-1, 1)$. Find the derivatives of the function in these directions.

33. Find the circulation of the field $F = \left(x^2 - y\right)i + 4zj + x^2k$ around the curve C in which the plane $z = 2$ meets the care $z = \sqrt{x^2 + y^2}$, counter clockwise as viewed from above.

34. (a) Verify the circulation form of the Green's theorem on the annular ring $R : h^2 \le x^2 + y^2 \le 1$, $0 < h < 1$ if $M = \dfrac{-y}{x^2 + y^2}, N = \dfrac{x}{x^2 + y^2}$.

(b) Find the work done by the conservative field $F = yzi + xzj + xyk = \nabla(xyz)$ along any smooth curve C joining the point $(-1, 3, 9)$ to $(1, 6, -4)$.

35. (a) A wheel of radius a rolls along a horizontal straight line. Find the parametric equations for the path traced by a point p on the wheel's circumference.

(b) Sketch the hyperbola $9x^2 - 16y^2 = 144$. Include the asymptotes and foci in your sketch.

$(2 \times 15 = 30)$