E 9705 (Pages : 4) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2014

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/ Food Science and Quality Control/Computer Maintenance and Electronics)

[2013 admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{y \to -5} \frac{y^2}{5-y}$.
2. State the chain rule for the derivative of a composite function of one variable.
3. Show that the derivative of a constant is zero.
4. Define absolute maximum of a function.
5. Define an increasing function.
6. State the Mean value theorem.
7. Define level surface of a function $f(x,y,z)$.
8. Find $\frac{\partial f}{\partial y}$ if $f(x,y) = x^y$.
9. Show that $\cosh^2 x - \sinh^2 x = 1$.
10. What is the period of $\cosh(x+iy)$?

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Prove the limit statement $\lim_{x \to 3}(3x-7) = 5$.

Turn over

12. Find $\frac{dy}{dx}$ as a function of t if $x = t + \frac{1}{t}, y = t - \frac{1}{t}$.

13. Find the slope of the circle $x^2 + y^2 = 2$ at the point (1, 1).

14. Find the absolute maximum value of $f(x) = x^2$ on [–2, 1].

15. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through (0, 2).

16. Find the value of C in the mean value theorem for $f(x) = x^2 + 2x - 1$ on [0, 1].

17. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = \frac{1}{x+y}$.

18. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$.

19. Draw a tree diagram and write a chain rule for $\frac{dz}{dt}$ where $z = f(x,y), x = g(t), y = h(t)$.

20. If $x = \cos\theta + i\sin\theta$, find $x^4 + \frac{1}{x^4}$ and $x^4 - \frac{1}{x^4}$.

21. Show that $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$.

22. Show that $\sinh^{-1} x = \log\left(x + \sqrt{x^2+1}\right)$.

(8 × 2 = 16)

Part C (Descriptive/Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 marks.

23. Show that $\lim_{x \to \infty} \frac{1}{x} = 0$ and $\lim_{x \to -\infty} \frac{1}{x} = 0$.

24. Find an equation for the tangent to the curve $y = x + \frac{2}{x}$ at (1, 3).

25. Find $\frac{d^2y}{dx^2}$ if $ax^2 + 2h\,xy + by^2 = 1$, where a, b, h are constants.

26. State and prove Rolle's theorem.

27. Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution.

28. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + y + z^2, x = \frac{r}{s}, y = r^2 + \log s, z = 2r$

29. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = \frac{2y}{y + \cos x}$.

30. Separate into real and imaginary parts of $\tan(x + iy)$.

31. Expand $\sin^6 \theta$ in a series of cosines of multiples of θ.

$(6 \times 4 = 24)$

Part D (Long Essay Questions)

Answer any **two** *questions.*
Each question carries **15** *marks.*

32. (a) Let $f(x) = x + 1$ and $\varepsilon = 0.01$. Find a $\delta > 0$ such that for all x with $0 < |x - 1| < \delta$ the inequality $|f(x) - 5| < \varepsilon$ holds.

(b) Show that $f(x) = |x|$ is differentiable except at $x = 0$.

(c) To position P (x, y) of a particle moving in the xy-plane is given by the equations : $x = \sqrt{t}, y = t, t \geq 0$. Identify the path traced by the particle and describe the motion.

33. (a) If the derivative $f'(x) = 0$ at each point x of an open interval (a, b), prove that $f(x) = c$ for all x in (a, b), where c is a constant.

(b) Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and decreasing.

(c) Suppose $f(0) = 5$ and $f'(x) = 2$ for all x. Must $f(x) = 2x + 5$ for all x ? Give reasons for your answer.

34. (a) Find all the second order partial derivatives of $f(x,y) = x^2 y + \cos y + y \sin x$.

(b) Find $\frac{dw}{dt}$ at $t = 3$, given that $w = \frac{x}{z} + \frac{y}{z}, x = \cos^2 t, y = \sin^2 t, z = \frac{1}{t}$.

Turn over

(c) Draw the free diagrams and chain rules for the derivatives $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for $z = f(x, y)\, x = g(t, s)$, $y = h(t, s)$.

35. (a) Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiples of θ.

(b) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$, prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$.

(c) Find the sum to infinity the series :

$$c\sin\alpha + \frac{c^2}{2!}\sin 2\alpha + \frac{c^3}{3!}\sin 3\alpha + \dots$$

(2 × 15 = 30)