G 420 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2010

Second Semester

INTEGRAL CALCULUS AND MATRICES

(Complementary course to Physics/Chemistry/Petrochemicals/Geology

B.Sc. Food Science and Quality Control B.Sc. Computer Maintenance and Electronics)

Time : Three Hours Total Weightage : 25

Part A (Objective Type Questions)

Answer **all** *the questions.*
Each bunch of 4 questions has weight 1.

I. 1 If f is integrable and $\int_1^2 f(x)\,dx = -4$, $\int_1^5 f(x)\,dx = 6$, find $\int_1^5 f(x)\,dx$.

2 Express the limit $\lim_{\|p\| \to 0} \sum_{k=1}^{n} \sqrt{4 - c_k^2}\,\Delta x_k$, where p is a partition of [0, 1] as a definite integral.

3 Find $\int_0^5 x^{3/2}\,dx$.

4 Find $\frac{d}{dx}\int_0^x \frac{dt}{1+t^2}$.

II. 5 Give an example of non-integrable function on [0, 1].

6 Check whether $f(x) = x^2 + x^3$ is an even function.

7 The circle $x^2 + y^2 = a^2$ is rotated about the x-axis. What is the solid of revolution?

8 Find the length of the curve $x = \cos t, y = t, + \sin t, 0 \le t \le \pi$.

III. 9 Define a continuously differentiable function.

10 Evaluate :

$$\int_0^3 \int_0^2 \left(4 - y^2\right) dy\,dx.$$

11 Consider the region bounded by the lines $x = 0$, $y = 2x$ and $y = 4$. Express the regions area as an integrated double integral.

Turn over

12 Change the Cartesian integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\,dx$ into an equivalent polar integral.

IV. 13 Find the rank of $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$.

14 What are the characteristic value of 3I, where I is the identity matrix of order 3 × 3 ?

15 What is the characteristic polynomial of the zero matrix of order 4 × 4 ?

16 Write the normal form of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

(4 × 1 = 4)

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17. Use the Max-Min inequality to find upper and lower bounds for the value of $\int_{0}^{1} \frac{dx}{1+x^2}$.

18. Show that if f is continuous on $[a, b]$, $a \neq b$ and if $\int_{a}^{b} f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

19. Evaluate $\int_{-1}^{1} r\sqrt{1-r^2}\,dr$.

20. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

21. Evaluate $\int_{0}^{3}\int_{0}^{2} \left(4 - y^2\right) dy\,dx$.

22. Find the average value of $f(x, y) = x \cos(xy)$ over the rectangle $R : 0 \leq x \leq \pi, 0 \leq y \leq 1$.

23. Check whether the matrices $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ are equivalent.

24. Find the characteristic polynomial of $\begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Calculate the area bounded by the x-axis and the parabola $y = 4 - x^2$.

26. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

27. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

28. Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$.

29. Sketch the region of integration for the integral $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2 - y^2}} xy\, dx\, dy$ and write an equivalent integral with the order or integration reversed.

30. Find all non-trivial solutions of :

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \\ x_1 - 4x_2 + 5x_3 &= 0 \end{aligned}$$

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Find the length of the curve $y = x^{3/2} ; 0 \le x \le 1$.

32. Evaluate the integral $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\, dy\, dx$.

33. Given $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. Use Cayley-Hamilton theorem to compute A^2, A^3, A^4, A^{-1}, A^{-2} and A^{-3}.

(2 × 4 = 8)