



22101915

QP CODE: 22101915

Reg No :

Name :

**B.Sc DEGREE (CBCS) SPECIAL SUPPLEMENTARY EXAMINATIONS,
MAY 2022**

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc
Computer Applications Model III Triple Main

2019 Admission Only

85DBEA41

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the union of two disjoint denumerable sets are denumerable?
2. Define absolute value function?
3. Find the supremum and infimum of $S = \left\{ \frac{1}{n} - \frac{1}{m} : m, n \in N \right\}$?
4. Let $I_n = \left(0, \frac{1}{n} \right)$, $n \in N$ Prove that $\bigcap_{n=1}^{\infty} I_n = \phi$
5. Define convergent and divergent sequences. Give examples.
6. If $X = (x_n)$ is a sequence of real numbers, (a_n) is a sequence of positive real numbers such that $\lim(a_n) = 0$ and if for some positive constant $C > 0$ and $m \in N$ we have $|x_n - x| < Ca_n$ for every $n \geq m$, then prove that $\lim(x_n) = x$.
7. If $X = (x_n)$ is a convergent sequence of real numbers and if $a \leq x_n \leq b$ for every n , prove that $a \leq \lim(x_n) \leq b$.
8. Find $\lim \left(\frac{(-1)^n}{n+2} \right)$.





9. Let (x_n) and (y_n) be two sequences of positive real numbers and suppose that for some positive real number L , $\lim(\frac{x_n}{y_n}) = L$, then prove that $\lim x_n = +\infty$ if and only if $\lim y_n = +\infty$.
10. State the Limit Comparison Test for the series.
11. Test for the convergence of the series $\sum \frac{2^n}{e^n}$.
12. Show that $\lim_{x \rightarrow 0} (\frac{1}{x^2}) = \infty$.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. State and prove
 - (a.) Arithmetic mean - Geometric Mean Inequality
 - (b.) Bernoulli's Inequality
14. Prove that If A, B are bounded sets then $\text{Sup}(A + B) = \text{Sup } A + \text{Sup } B$ where $A + B = \{a + b : a \in A, b \in B\}$
15. Prove that for any real number $a > 0$, there exists a sequence (s_n) of real numbers that converges to \sqrt{a} .
16. State and prove Monotone Subsequence Theorem.
17. Let $X = (x_n)$ be the sequence defined as $x_1 = 1, x_2 = 2$ and $x_n = \frac{x_{n-2} + x_{n-1}}{2}$ for $n > 2$.
Prove that $\lim X = \frac{5}{3}$.
18. Prove that if $\sum x_n$ is a convergent series then any series obtained from it by grouping terms is also convergent. What about the converse of this?
19. If (x_n) is a monotone convergent sequence and $\sum y_n$ is convergent, then establish the convergence of $\sum x_n y_n$
20. Show that $\lim_{x \rightarrow 2} \frac{x^3 - 4}{x^2 + 1} = \frac{4}{5}$ using $\varepsilon - \delta$ definition.
21. If $A \subseteq \mathcal{R}$ and $f : A \rightarrow \mathcal{R}$ has a limit at $c \in \mathcal{R}$, then prove that f is bounded on some neighborhood of c .

(6×5=30)





Part C

Answer any **two** questions.

Each question carries **15** marks.

22. (a.) State and Prove Nested interval property?
 (b.) Prove that the set of real numbers is not countable?
23. (a) State and prove Monotone Convergence Theorem.
 (b) Prove that (x_n) is divergent, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ for every $n \in \mathbf{N}$.
24. Test the convergence and absolute convergence of the following series.

- $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n^2+1)}$
- Whose n th term is $\frac{n^n}{(n+1)^{n+1}}$

25. (a) Let $A \subseteq \mathcal{R}$, $f, g : A \rightarrow \mathcal{R}$, and let $c \in \mathcal{R}$ be a cluster point of A , Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim_{x \rightarrow c} f = \infty$, then $\lim_{x \rightarrow c} g = \infty$.
- If $\lim_{x \rightarrow c} g = -\infty$, then $\lim_{x \rightarrow c} f = -\infty$.

(b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(c) Evaluate the limit or show that it does not exist " $\lim_{x \rightarrow 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)

