E 5860 (Pages : 3) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and Model II and B.Sc. Computer Application)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. When does the positive term geometric series $1 + r + r^2 + \ldots$ converge ? Diverge ?
2. Is the series $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{4^3} + \cdots$ convergent ? Why ?
3. Define an alternating series.
4. Define removable discontinuity.
5. State the Intermediate Value Theorem.
6. Define partition of $[a, b]$ and a refinement of a partition.
7. State Darboux's theorem.
8. Let $f(x) = \frac{1}{2}$ for all x in $[0, 1]$. Is f Riemann integrable on $[0, 1]$? Why or why not ?
9. Define uniform convergence of a sequence of functions.
10. State Weierstress's M-test.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. If $\sum u_n$ is convergent, prove that $\lim\limits_{n \to \infty} u_n = 0$.
12. Test the convergence of the series : $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$.
13. State Raabe's test.

14. Prove or disprove : Every convergent series is absolutely convergent.

15. If $[x]$ denotes the largest integer $\leq x$, discuss the continuity at $x = 4$ for the function $f(x) = x - [x], x \geq 0$.

16. Prove that a function which is uniformly continuous on an interval is continuous on that interval.

17. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

18. Explain upper and lower integrals of a bounded function f on $[a, b]$.

19. If f is integrable on $[a, b]$, prove that $|f|$ is integrable on $[a, b]$.

20. Let f be a bounded function on $[a, b]$. Prove that for any two partitions P_1, P_2, $L(P_1, f) \leq U(P_2, f)$.

21. Show that the sequence $\{f_n\}$, where $f_n(x) = \dfrac{1}{x+n}$ is uniformly convergent on $[0, 1]$.

22. Show that $\Sigma \dfrac{\cos n\theta}{n^p}$, $p > 1$ is uniformly convergent for all real values of θ.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Prove that the positive term series $\Sigma \dfrac{1}{n^p}$ is convergent for $p > 1$.

24. State and prove Cauchy's root test.

25. Show that the series $x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$ converges absolutely for all values of x.

26. If a function f is continuous on $[a, b]$, prove that it attains its bounds at least once in $[a, b]$.

27. Show that the function f defined by $f(x) = \sin\dfrac{1}{x}$ for $x \neq 0$ and $f(0) = 0$ is not uniformly continuous on $[0, \infty]$.

28. If f is integrable on $[a, b]$, prove that f^2 is integrable on $[a, b]$.

29. Let f be defined by $f(x) = \dfrac{1}{2^n}$ for $\dfrac{1}{2^{n+1}} < x < \dfrac{1}{2^n}$ $(n = 0, 1, 2, \ldots)$ and $f(0) = 0$. Compute $\int_0^1 f\, dx$.

30. State and prove Cauchy's criterion for uniform convergence of a sequence of functions.

31. Show that the sequence $\{f_n\}$ where $f_n(x) = \dfrac{nx}{1+n^2x^2}$ is not uniformly convergent on any interval containing zero.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 marks.

32. (a) State and prove D'Alembert's ratio test.

    (b) Test for convergence the series $\sum \frac{n^2 - 1}{n^2 + 1} x^n, x > 0$.

33. (a) Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

    (b) Is the above property true if the interval is not closed ? Justify your answer.

34. (a) Prove that a bounded function f is integable on $[a, b]$ if and only if for every $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

    (b) State and prove the Fundamental theorem of Calculus.

35. (a) Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$ is uniformly convergent on $[0, k]$, $k < 1$ and only pointwise convergent on $[0, 1]$.

    (b) State Dirichlet's test for uniform convergence of a series.

    (c) Prove that the series $\sum (-1)^n \frac{x^2 + n}{n^2}$, converge uniformly on every bounded interval, but does not converge absolutely for any value of x.

(2 × 15 = 30)