



**QP CODE: 23105584**

**Reg No** : .....

**Name** : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,  
MARCH 2023**

**Sixth Semester**

**CORE COURSE - MM6CRT01 - REAL ANALYSIS**

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc  
Computer Applications Model III Triple Main

2017 Admission Onwards

7566B152

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give an example of a function defined on  $\mathbb{R}$  which is discontinuous only at  $x=0$ .
2. Give an example of a discontinuous function defined on a closed interval  $C$  but not bounded on  $C$ .
3. Define jump at  $c$  of the function  $f : [a, b] \rightarrow \mathbb{R}$  where  $a < c < b$ .
4. Let  $f, g$  are differentiable functions, then prove that  $f + g$  is also differentiable?
5. Prove that if  $f : \mathbb{R} \rightarrow \mathbb{R}$  is an odd function ( $f(-x) = -f(x) \forall x$ ), then its derivative is an even function?
6. Using the chain rule, find the derivative of  $f^n(x)$ , where  $f : I \rightarrow \mathbb{R}$  is differentiable and  $n \geq 2, n \in \mathbb{N}$
7. How Riemann integral is related with areas of regions in  $\mathbb{R}^2$ .
8. Let  $f : [a, b] \rightarrow \mathbb{R}$  and  $C \in \mathbb{R}$ . Show that if  $\phi$  is an antiderivative of  $f$  on  $[a, b]$  then  $\phi + C$  is also an antiderivative of  $f$  on  $[a, b]$
9. Give an example of a function on  $[a, b]$  which is bounded but not Riemann integrable.
10. Evaluate  $\lim x^2 e^{-nx}$ .





11. State a sufficient condition to guarantee that the limit of a sequence of continuous functions is continuous.
12. Give an example of a sequence of differentiable functions  $(f_n)$  which converges point wise to a function  $f$  on  $[a, b]$ , but  $f$  is not differentiable on  $[a, b]$ .

(10×2=20)

### Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Define Thomae's function on  $(0, \infty)$  and show that it is continuous precisely at the irrational points in  $(0, \infty)$ .
14. Let  $A, B \subseteq \mathbb{R}$ , let  $f : A \rightarrow \mathbb{R}$  be continuous on  $A$ , and let  $g : B \rightarrow \mathbb{R}$  be continuous on  $B$ . If  $f(A) \subseteq B$ , show that the composite function  $g \circ f : A \rightarrow \mathbb{R}$  is continuous on  $A$ .
15. Show that every continuous function on a closed bounded interval  $I$  is uniformly continuous on  $I$ .
16. State and Prove Mean value theorem?
17. State and prove the first derivative test for extrema?
18. Evaluate the limit  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}, x \in (0, \infty)$
19. Prove that if  $f : [a, b] \rightarrow \mathbb{R}$  is a continuous function then  $f \in \mathcal{R}[a, b]$ .
20. Evaluate  $\int_1^4 \frac{\sqrt{1+\sqrt{t}}}{\sqrt{t}} dt$ .
21. Show that the sequence  $(x^2 e^{-nx})$  converges uniformly on  $[0, \infty)$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. (a) State and prove Uniform Continuity Theorem..  
(b) State and prove Continuous Extension Theorem.
23. (a.) State and Prove L'Hospital's Rule I  
(b.) Using this, find the following





$$(i.) \lim_{x \rightarrow 0+} \frac{\tan x}{x}$$

$$(ii.) \lim_{x \rightarrow 0+} \frac{\log(x+1)}{\sin x}, x \in (0, \frac{\pi}{2})$$

24. (a) Suppose that  $f : [a, b] \rightarrow \mathbb{R}$  and that  $f(x) = 0$ , except for a finite number of points  $c_1, c_2, \dots, c_n$  in  $[a, b]$ . Prove that  $f \in \mathcal{R}[a, b]$  and  $\int_a^b f = 0$ .

- (b) If  $g \in \mathcal{R}[a, b]$  and if  $f(x) = g(x)$  except for a finite number of points in  $[a, b]$ , prove that  $f \in \mathcal{R}[a, b]$  and that  $\int_a^b f = \int_a^b g$ .

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function  $f : [a, b] \rightarrow \mathbb{R}$ .

- (b) Check the Riemann integrability of Dirichlet function.

(2×15=30)

