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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2011

Third Semester

Core Course-3—CALCULUS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type Questions)

Answer all questions.

Each bunch of four questions carries weight 1.

- I. 1 Value of  $D^n(ax+b)^n$  is ———.
- 2 The Mclaurin's series for  $e^x$  is ———.
- 3 The curve  $y = f(x)$  is ——— at  $(c, f(c))$  if  $f''(c) < 0$ .
- 4 The curvature of a straight line at every point is ———.
- II. 5 The evolute of a curve is the envelope of its ———.
- 6 Find  $\frac{\partial f}{\partial x}$  if  $f(x, y, z) = e^{-xyz}$ .
- 7 If  $W = f(x, y)$ ,  $x = g(r, s)$ ,  $y = h(r, s)$  then  $\frac{\partial W}{\partial r} =$  ———.
- 8 Write the discriminant of the function  $f(x, y)$ .
- III. 9 Let  $f$  be continuous on  $[-a, a]$ . If  $f$  is even then  $\int_{-a}^a f(x) dx = \dots$
- 10 Find the volume of the solid of cross-sectional area  $A(x) = 2x$  from  $x = 0$  to  $x = 2$ .
- 11 Write the formula for the length of the curve  $x = f(t)$ ,  $y = g(t)$ ,  $a \leq t \leq b$ .
- 12 Write the surface area formula in the differential form.
- IV. 13 Express the area bounded in the plane by the  $x$ -axis, the lines  $y = x$  and  $x = 1$ , as a double integral.
- 14 Write the formula for the area of a bounded region  $R$  in the polar co-ordinate plane.
- 15 Define the volume of a closed bounded region  $D$  in space using triple integral.
- 16 Write the equation of a sphere of radius 5 with centre at origin, in spherical co-ordinates.

(4 × 1 = 4)

Turn over

## Part B (Short Answer Type Questions)

Answer any five questions.

Each question carries weight 1.

- 17 Differentiate the equation  $n$  times with respect to  $x$ :  $(1-x^2)y_2 - xy_1 - y = 0$ .
- 18 Find the asymptotes parallel to the co-ordinate axes of the curve  $(x^2 + y^2)x - ay^2 = 0$ .
- 19 Find  $\frac{dw}{dt}$  at  $t = 0$  if  $w = xy + z$ ,  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$ .
- 20 Find the local extreme values of  $f(x, y) = x^2 + y^2$ .
- 21 Evaluate  $\int_0^1 t^3 (1+t^4)^3 dt$ .
- 22 Find the volume of the sphere generated by rotating the circle  $x^2 + y^2 = 1$  about the  $x$ -axis.
- 23 Find the area of the region enclosed by the curve  $y = x^2$  and the line  $y = x + 2$ , by double integrals.
- 24 Evaluate the cylindrical co-ordinate integral  $\int_0^{2\pi} \int_0^2 \int_0^2 z \, dz \, r \, dr \, d\theta$ .

(5 × 1 = 5)

## Part C (Short Essay Questions)

Answer any four questions.

Each question carries weight 2.

- 25 Find the points of inflexion on the curve  $y = (\log x)^3$ .
- 26 Find the evolute of the parabola  $y^2 = 4ax$ .
- 27 Find the greatest and smallest values of that the function  $f(x, y) = xy$  takes on the ellipse  $\frac{x^2}{8} + \frac{y^2}{2} = 1$ .
- 28 Find the length of the curve  $y = x^{3/2}$  from  $x = 0$  to  $x = 4$ .
- 29 Find the area of the surface that is generated by revolving the portion of the curve  $y = x^3$  between  $x = 0$  and  $x = 1$  about the  $x$ -axis.
- 30 Evaluate  $\int_0^1 \int_{\sqrt{1-y^2}}^1 xy \, dx \, dy$  by expressing it as an equivalent integral with the order of integration reversed.

(4 × 2 = 8)

## Part D (Essay Questions)

Answer any two questions.  
Each question carries weight 4.

- 31 If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ , where  $-1 < x < 1$  and  $-\frac{\pi}{2} < \sin^{-1} x < \frac{\pi}{2}$ , show that

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0.$$

Assuming that  $y$  can be expanded in ascending powers of  $x$  in the form  $a_0 + a_1x + \dots + a_nx^n + \dots$ , prove that  $(n+1)a_{n+1} = na_{n-1}$ , and hence obtain a general term of the expansion.

- 32 The region bounded by the curve  $y = x^2 + 1$  and the line  $y = -x + 3$  is revolved about the  $x$ -axis to generate a solid. Find the volume of the solid.

- 33 Find the average value of  $F(x, y, z) = (1 - x^2 - y^2 - z^2)^{-1/2}$  over the sphere  $x^2 + y^2 + z^2 = 1$ .

(2 × 4 = 8)