



23105587

QP CODE: 23105587

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,****MARCH 2023****Sixth Semester****CORE COURSE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES**

Common for B.Sc Mathematics Model I &amp; B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

1552A4CE

Time: 3 Hours

Max. Marks : 80

**Part A***Answer any **ten** questions.**Each question carries **2** marks.*

1. Define a Graph.
2. When will you say that two graphs are isomorphic?
3. Draw all non-isomorphic complete bipartite graphs with atmost 4 vertices.
4. What do you mean by an underlying simple graph?
5. Define a bridge. Write the relation between order and size of a connected graph.
6. Define Cut vertex, If  $P_n$  is a path of length  $n$ , where  $n \geq 3$ , Then identify the cut vertices of  $P_n$ .
7. Define Euler graph. Give one example.
8. Define a maximal non Hamiltonian graph. Give an example.
9. Prove that in a metric space  $X$ , finite intersection of open sets is open.
10. Define Cantor set.
11. Define limit of a sequence in a metric space.
12. Define complete metric space. Give an example.

(10×2=20)

**Part B***Answer any **six** questions.**Each question carries **5** marks.*

13. Let  $G$  be a simple graph with  $n$  vertices, where  $n \geq 2$ . Prove that  $G$  has two vertices  $u$  and  $v$  with  $d(u) = d(v)$ .





14. Define adjacency matrix of a graph. Draw the graph whose adjacency matrix is 
$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$
. What can you say about the graph if all the entries of the main diagonal are zero?
15. a) Let  $G$  be an acyclic graph with  $n$  vertices and  $k$  connected components, then prove that  $G$  has  $n - k$  edges.  
b) Justify above result using a graph with 8 vertices.
16. Prove that a graph  $G$  is connected if and only if it has a spanning tree.
17. If  $G$  is a simple graph with  $n$  vertices, where  $n \geq 3$ , and the degree  $d(v) \geq \frac{n}{2}$  for every vertex  $v$  of  $G$ , Then prove that  $G$  is Hamiltonian.
18. Prove that  $A$  is open if and only if  $A = \text{int } A$ .
19. a) Define closure of a set in a metric space  $X$ .  
b) Prove that  $A$  is closed if and only if  $\bar{A} = A$ .
20. Is limit of a sequence, a limit point of the underlying set? Justify with suitable examples.
21. Let  $X$  and  $Y$  be metric spaces and  $f$  a mapping of  $X$  into  $Y$ . Prove that  $f$  is continuous if and only if  $f^{-1}(G)$  is open in  $X$  whenever  $G$  is open in  $Y$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. (a) State and prove First theorem of graph theory.  
(b) Prove that in any graph  $G$  there is an even number of odd vertices.  
(c) Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.
23. a) State and prove Whitney's theorem for 2- connected graphs.  
b) Let  $u$  and  $v$  be two vertices of the 2- connected graph. Then prove that there is a cycle passing through both  $u$  and  $v$ .
24. a) Define a metric space. Give an example of a metric on  $\mathbf{R}$ .  
b) Let  $X$  be the collection of all bounded real valued functions on  $[0,1]$ . Prove that  $d$  defined  $d(x,y) = \|f - g\|$ , where  $\|f\| = \sup\{|f(x)| : x \in [0,1]\}$  is a metric on  $X$ .
25. (a) State and prove Cantor's Intersection Theorem.  
(b) Let  $X$  be a complete metric space and  $Y$  a subspace of  $X$ . Prove that  $Y$  is complete if and only if  $Y$  is closed.

(2×15=30)

