



QP CODE: 24001057



24001057

Reg No :

Name :

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024

Sixth Semester

CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

A66A3E10

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define Interior point and Boundary point in terms of neighbourhood
2. Find $f'(z)$ where $f(z) = \frac{z-1}{2z+1}$ where $z \neq -\frac{1}{2}$
3. If $u+iv$ is analytic, then under what condition will $v+iu$ be analytic
4. Find i^i and its principal value
5. Separate the real and imaginary parts of $\sinh z$.
6. Define Simple closed curve.
7. What is the value of $\int_C (z-1)dz$ where C is the line segment $z=x$, $0 \leq x \leq 2$.
8. Define simply connected and multiply connected domain.
9. Define the limit of an infinite sequence of complex numbers.
10. With the aid of the identity $\cos z = -\sin\left(z - \frac{\pi}{2}\right)$, expand $\cos z$ into a Taylor series about the point $z_0 = \frac{\pi}{2}$
11. State Cauchy's Residue Theorem.
12. Prove that if the improper integral over $-\infty < x < \infty$ exists, then its Cauchy Principal Value exists.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. If a function is analytic, Show that it is independent of \bar{z}





14. Prove that $|\exp(-2z)| < 1$ if and only if $\operatorname{Re}(z) > 0$
15. Find where $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$ is analytic
16. Evaluate $\int_C \frac{1}{z^2+2z+2} dz$ where C is the circle $|z|=1$.
17. Evaluate $\int_C \frac{\sinh z}{(2z - z^2)^2}$ Where C is the circle $|z| = 1$ oriented counterclockwise
18. State and prove Cauchy's inequality.
19. Use Maclaurin's series expansion of $\sin z$ to obtain such a series for $\cos z$
20. Using residues, evaluate $\int_C e^{(\frac{1}{z^2})} dz$ where C is the unit circle about the origin.
21. State the characterization of poles of order m of a complex function $f(z)$ and the formula for residue at z_0 of the poles of order m . Find the residue at $z = i$ of $f(z) = \frac{z^3+2z}{(z-i)^3}$.
(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. a) State and prove the sufficient condition for a function $f(z)$ to be differentiable.
b) Show that the function $f(z) = \ln(|z|) + i \operatorname{Arg}(z)$ is analytic on its domain of definition and $f'(z) = \frac{1}{z}$
23.
 - Prove that any polynomial of degree n has at least one zero
 - State and Prove Liouville's theorem
24. a) Derive the Laurent series expansion of $\frac{e^z}{(z+1)^2}$ in terms of $z+1$, if $0 < |z+1| < \infty$
b) Let $f(z) = \frac{1}{(z-i)^2}$. Use Laurent series expansion to prove that $\int_C \frac{dz}{(z-i)^{-n+3}} = 2\pi i, n = 2$
c) Show that for $0 < |z-1| < 2$ $\frac{z}{(z-1)(z-2)} = \frac{-1}{2(z-1)} - 3 \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+1}}$
25. Define the Removable singular points, essential singular points and a pole of order m , of a complex function with examples. Verify the examples with their series representations.

(2×15=30)

