E 4221 (Pages : 4) Reg. No..............................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2016

First Semester

Core Course—FOUNDATION OF MATHEMATICS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 mark.

1. Define the power set of a set.

2. Why is f not a function from R to R if $f(x) = \frac{1}{x-1}$?

3. Define a reflexive relation.

4. What is the equivalence class of 6 for the relation congruence modulo 5 ?

5. Define a partial ordering.

6. What is the contra positive of the statement. "If it is raining, then the home team wins".

7. What does it mean for two propositions to be logically equivalent ?

8. Find the number of divisors of 1000.

9. Give a test as to the divisibility of a number by 7.

10. State Fermat's theorem.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. What is the empty set ? Show that the empty set is a subset of every set.

12. Determine whether the function $f : \mathrm{R} \to \mathrm{R}$ defined by $f(x) = -3x + 4$ is a surjection.

Turn over

13. If $S = \{1, 3, 5, 7\}$ find the value of the sum $\sum_{i \in S} i^2$.

14. Define a relation on a set. How many relations are there on the set $\{1, 2, 3, 4\}$?

15. Draw the directed graph representing the relation, $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (4, 4)\}$ on the set $\{1, 2, 3, 4\}$.

16. List the ordered pairs in the equivalence relation produced by the partition

$$A_1 = \{0, 1\}, A_2 = \{2, 3\}, A_3 = \{4, 5\} \text{ of the set } S = \{0, 1, 2, 3, 4, 5\}.$$

17. Construct the truth table for the biconditional $p \leftrightarrow q$.

18. Write the negations of the statements

$$\forall x \left(x^2 > x\right) \text{ and } \exists x \left(x^2 = 2\right).$$

19. Prove or disprove that the product of two irrational numbers is irrational.

20. Prove that $n(n+1)(n+2)$ is divisible by 6 for any positive integer n.

21. If $a_1 \equiv b_1, (\bmod\ n)$ and $a_2 \equiv b_2 \ (\bmod\ n)$, prove that $a_1 a_2 \equiv b_1 b_2 \ (\bmod\ n)$.

22. Find the $g\ c\ d$ of 162 and 138.

$(8 \times 2 = 16)$

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. If A and B are subsets of a universal set U, prove that $A \subseteq B$ if and only if $\bar{B} \subseteq \bar{A}$.

24. Prove that the set of all odd positive integers is countable.

25. Determine the number of reflexive relations on a set with n elements.

26. Let $m > 1$ be a positive integer. Prove that the relation congruence modulo m is an equivalence relation on the set of integers.

27. Suppose the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3 and 4. Write the proposition (i) $\exists x\, P(x)$; and (ii) $\forall x \neg P(x)$ using disjunctions, conjunctions and negations.

28. Express the statement $\lim_{x \to a} f(x) = L$ using quantifiers.

29. Prove that every square number is one of the forms $5n, 5n \pm 1$.

30. Define Eulers function $\phi(n)$. If $n \geq 2$, prove that the sum of positive integers less than n and prime to n is $\frac{1}{2} n\, \phi(n)$.

31. If p is a prime number, prove that $\underline{|p-1} + 1$ is divisible by p.

(6 × 4 = 24)

Part D (Long Essay Type Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) For all sets A, B and C prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

(b) Define the ceiling function and draw its graph.

(c) Prove that the set of all real numbers between 0 and 1 is uncountable.

33. (a) Let R be an equivalence relation on a set A. For elements a and b of A, prove that the following statements are equivalent :

(i) $a\, R\, b$.

(ii) $[a] = [b]$.

(iii) $[a] \cap [b] = \phi$.

(b) Obtain the sets in the partition of the integers arising from congruence modulo 5.

(c) Draw the Hasse diagram of the poset $(P(S), \subseteq)$, where $S = \{a, b, c\}$. Is this poset a lattice ? Justify your answer.