



QP CODE: 24019463



24019463

Reg No :

Name :

**B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE
EXAMINATIONS, MAY 2024**

Second Semester

**Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY,
TRIGONOMETRY AND DIFFERENTIAL CALCULUS**

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

0835921B

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Derive the condition that the line $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Show that two tangents can be drawn from any point to a hyperbola.
3. Find the orthoptic locus of the parabola $y^2 = 4ax$.
4. Derive the equation of chord of contact of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
5. Find the cartesian coordinate corresponding to the polar coordinate (3,0).
6. Find the equation for a circle centered at the pole. Give an example.
7. Prove that $\sin(x+y) = \sin x \cos y + \cos x \sin y$.
8. Define Hyperbolic functions.
9. Factorize $x^9 + 1$.
10. Find the n^{th} derivative of $\log_e(ax + b)$.
11. Using Leibnitz's theorem find the n^{th} derivative of $e^x \log x$.
12. State L' Hospital's rule of limits.

(10×2=20)





Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Prove that the tangent at the extremity of a diameter of a parabola is parallel to the system of chords it bisects.
14. The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$, then the middle point of QR lies in the parabola $l(y^2 - 4ax) + 2a(lx + my + n) = 0$.
15. Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2a^6 - x^2b^6 = (a^2 + b^2)^2x^2y^2$.
16. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal. Also find the minimum angle.
17. If the normal at P on a conic meets the axis in G, then $SG = e \cdot SP$, where S is the focus.
18. If $\sin(A + iB) = x + iy$, show that
 - (i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$
 - (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
19. Sum the series $1 + c \cosh \alpha + c^2 \cosh 2\alpha + \dots + c^{n-1} \cosh(n-1)\alpha$, where c is less than unity.
20. If $y = [x + \sqrt{1 + x^2}]^m$, show that $(1 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - m^2y = 0$
21. Find $\lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right]^{\frac{1}{x}}$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Prove that
 - (a) The tangents at the extremities of a diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are parallel to the diameter conjugate to it.
 - (b) The tangents at the extremities of a chord of an ellipse will intersect on the diameter conjugate to the diameter parallel to the given chord.
 - (c) The eccentric angles of the ends of a pair of conjugate diameters differ by a right angle.





23. If two conics have a common focus, show that two of their common chords pass through the point of intersection of their directrices.

24. Sum the series

(i) $c \cos \alpha - \frac{1}{3} c^3 \cos(\alpha + 2\beta) + \frac{1}{5} c^5 \cos(\alpha + 4\beta) - \dots$

(ii) $\sin \alpha \sin \beta + \frac{1}{2} \sin 2\alpha \sin 2\beta + \frac{1}{3} \sin 3\alpha \sin 3\beta + \dots$

25. (a) Show that the n^{th} derivative of $y = \frac{1}{1+x+x^2+x^3}$ is $\frac{1}{2}(-1)^n n! \sin^{n+1} \theta [\sin(n+1)\theta - \cos(n+1)\theta + (\sin \theta + \cos \theta)^{-n-1}]$ where $\theta = \cot^{-1} x$.

(b) Find the n^{th} derivative of $\frac{1}{(x-1)^2(x-2)}$.

(2×15=30)

