QP CODE: 22101059

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022

Sixth Semester

CORE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

50414609

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Show that $\lim\limits_{z\to 0} \frac{z}{\bar{z}}$
2. Find an analytic function with real part x^2y.
3. Find the real part of e^{-3z}?
4. Find all the roots of tan z=1
5. Evaluate $\tanh^{-1}(1-i)$.
6. Evaluate $\int_0^{\frac{\pi}{6}} e^{i2t}dt.$
7. What is the value of $\int_{|z|=1}(z^2+4)dz.$
8. Evaluate $\int_C \frac{e^z}{z-2}dz,$ C is the circle |z|=3.
9. Define the convergence of an infinite series of complex numbers.
10. *Show that when* $0<|z|<4$, $\frac{1}{4z-z^2}=\frac{1}{4z}+\sum\limits_{n=0}^{\infty}\frac{z^n}{4^{n+2}}$
11. Define isolated singular points and show that $z=0$ is not an isolated singular point of Log z.
12. Show that the existence of Cauchy Principal Value does not imply the existence of $\int_{\infty}^{-\infty} f(x)dx$

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that the composition of two continuous functions is continuous

14. Prove that the function f(z) = $\sqrt{|xy|}$ is not analytic at the origin , eventhough CR Equations are satisfied at the origin

15. Show that $Re[\log(z-1)] = \frac{1}{2}\ln[(x-1)^2 + y^2], z \neq 1$

16. Evaluate $\int_C \frac{\sinh z}{(2z - z^2)^2}$ Where C is the circle $|z| = 1$ oriented counterclockwise

17. If f(z) is analytic within and on a circle C given by $|z - z_0| = R$ and if $|f(z)| \leq M$ for every z on C,Prove that $|f^n(z_0)| \leq M\frac{n!}{R^n}$

18. State and prove maximum modulus principle.

19. *Find the Maclaurin series expansion of the function* $f(z) = \frac{z}{z^4+9}$ *and the interval in which the expansion is valid*

20. Define the essential singular points of a complex function with example. Verify the example with its series representations

21. State the characterization of poles of order of $f(z) = \frac{z^3+2z}{(z-i)^3}.m$ of a complex function $f(z)$ and the formula for residue at z_0 of the poles of order m. Find the residue at $z = i$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) State and prove the sufficient condition for a function f(z) to be differentiable.
b) Show that the function f(z) =ln(|z|)+i Arg(z) is analytic onits domain of definition and f'(z)=$\frac{1}{z}$

23. Evaluate $\int_c f(z)dz, \text{where } f(z) = \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1+i and i, the orientation of C being in the counter clockwise direction.

24. *a) Represent the function* $f(z) = \frac{z+1}{z-1}$ *by its Laurent series in the domain* $1 <| z |< \infty$
b) Show that for $0 <| z-1 |< 2, \frac{z}{(z-1)(z-2)} = \frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+1}}$

25. State and prove Cauchy's Residue Theorem. Using the theorem, evaluate $\int_C z^2 e^{(\frac{1}{z})} dz$, where C is the circle $|z| = 3$

(2×15=30)