E 1404 **(Pages : 5)** **Reg. No..............................**

Name..................................

C.B.C.S.S. – B.Sc. DEGREE EXAMINATION, APRIL 2011

Second Semester

Complementary Course—OPERATIONS RESEARCH–DUALITY, TRANSPORTATION AND ASSIGNMENT PROBLEMS

(For B.Sc. Mathematics Model–II)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of 4 *questions has weight* 1.

I. 1 Consider the Linear programming problem :

Minimize $\sum_{j=1}^{n} c_j x_j$ with respect to the constraints $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$ and $x_j \geq 0$ for $j = 1, 2, ..., n$.

Write the constraints of the dual LP problem.

2 Assume that in the optimal solutions of the primal and dual, a primal variable x_j is positive. What is the value of the corresponding dual slack variable y_{m+j} ?

3 What is the relation between dual of the dual and primal ?

4 If in the primal LP problem, the j^{th} variable $Xj \geq 0$, then what can we say about the j^{th} constraint ?

II. 5 Assume that there are m sources and n sinks in a transportation problem. What is the order of the transportation matrix ?

6 Consider a transportation problem with m sources and n sinks. In a basic solution of this transportation problem, what is the maximum number of variables having non-zero values ?

7 Give an example of a triangular set of equations.

8 How many basic variables are there in a balanced transportation problem with m sources and n destinations ?

III. 9 Define a loop in a transportation array.

10 Define a transportation problem which is unbalanced.

11 Suppose that we have solved an unbalanced transportation problem by adding a fictitious source. In the optimal solution of the new balanced transportation problem, how we interpret the supply from the additional source with respect to the new problem ?

Turn over

12 In a flow net model of a transportation problem, what is the difference between the inflow and outflow at a sink ?

IV. 13 Which of the following is True :—

(i) Assignment problem is a particular form of a transportation problem.

(ii) Transportation problem is a particular form of an assignment problem.

14 Consider an assignment problem with n workers. How many basic variables are there in a basic feasible solution for this problem ?

15 State the generalised transportation problem.

16 What is the formula that we use to determine the cost coefficients of non–basic variables in a transportation algorithm ?

$(4 \times 1 = 4)$

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Write the dual of the following LP problem :—

Minimize $6x_1 + 3x_2$

subject to

$$\begin{aligned} 3x_1 + 4x_2 + x_3 &\geq 5 \\ 6x_1 - 3x_2 + x_3 &\geq 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

18 Write a short note on advantages of duality in LP problems.

19 Describe the transportation matrix through an example.

20 Test whether the following six variables shown in the following table from a triangular set of equations, where $m = 3$, $n = 4$.

x_{11}	x_{12}		
x_{21}	x_{22}		
		x_{33}	x_{34}

21 Describe a method to solve an unbalanced transportation problem.

22 Give an example of a transportation-cum-trans-shipment problem.

23 Write the objective function and constraints in an assignment problem.

24 Prove that in an assignment problem with n workers $n - 1$ basic variables are zero.

$(5 \times 1 = 5)$

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Write the dual of the following LP problem :—

Maximize $f = 3x_1 + 2x_2 + x_3 + 4x_4$

subject to

$$2x_1 + 2x_2 + x_3 + 3x_4 \le 20$$
$$3x_1 + x_2 + 2x_3 + 2x_4 \le 20$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

Solve the dual to find the optimal solution of the primal.

26 Prove that the value of the objective function $f(x)$ for any feasible solution of the primal is not less than the value of the objective function $\phi(y)$ for any feasible solution of the dual.

27 Solve the following transportation problem for minimum cost starting with the degenerate solution $x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_{43} = 60$

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

28 Solve the following problem of transportation with trans-shipment with sources S_1, S_2 sinks D_1, D_2 and junction J for minimum cost.

		S_1	S_2	J	D_1	D_2
Transportation cost		4	3	1	3	5
Capacity		60	40	—	35	45
	S_1	—	4	3	10	5
Transportation cost	S_2	4	—	2	5	6
	J	4	2	—	8	7
	D_1	11	4	6	—	4
	D_2	5	7	5	4	—

29 Write the transportation algorithm.

30 Find the minimum cost solution for the 5×5 assignment problem whose cost coefficients are given below :

	1	2	3	4	5
1	-2	-4	-8	-6	-1
2	0	-9	-5	-5	-4
3	-3	-8	-9	-2	-6
4	-4	-3	-1	0	-3
5	-9	-5	-8	-9	-5

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 The manager of an agricultural farm of 80 hectares learns that for effective protection against insects, he should spray at least 15 units of chemical A and 20 units of chemical B per hectare. Three brands of insecticides are available in the market which contain these chemicals. One brand contains 4 units of A and 8 units of B per kg and costs Rs. 5 per kg, the second brand contains 12 and 8 units respectively and costs Rs. 8 per kg, and the third contains 8 and 4 units respectively and costs Rs. 6 per kg. It is also learnt that more than 2.5 kg per hectare of insecticides will be harmful to the crops. Determine the quantity of each insecticide he should buy to minimize the total cost for the whole farm.

32 Food bags have to be lifted by three different types of aircraft A_1, A_2, A_3 from an airport and dropped in flood affected villages V_1, V_2, V_3, V_4, V_5. The quantity of food that can be carried in one trip by aircraft A_i to village V_j is given in the following table. The total number of trips that A_i can make in a day is given in the last column. The number of trips possible each day to village V_j is given in the last row. Find the number of trips each aircraft should make on each village so that the total quantity of food transported in a day is maximum.

	V_1	V_2	V_3	V_4	V_5	
A_1	10	8	6	9	12	50
A_2	5	3	8	4	10	90
A_3	7	9	6	10	4	60
	100	80	70	40	20	

33 Four seminars are to be organised for a class in a week, Monday through Friday, such that not more than one seminar is held per day and the number of students who cannot attend is kept at the minimum. It is estimated that the number of students who are not free to attend a seminar on a particular day is as follows. Also seminar S_3 cannot be held on Tuesday. Find the optimal schedule of seminars to Maximize attendance.

	S_1	S_2	S_3	S_4
Monday	60	20	50	40
Tuesday	40	30	10	30
Wednesday	30	20	60	20
Thursday	20	30	30	30
Friday	10	30	10	20

$(2 \times 4 = 8)$