E 2450 (Pages : 4) Reg. No..............................

Name..............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2016

Second Semester

Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for B.Sc. Mathematics Model I, Model II and B.Sc. Computer Application)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*

Each question carries **1** *mark.*

1. Write the equation of the tangent at 'θ' on the hyperbola $\frac{x^2}{a^2} - \frac{y_2}{b^2} = 1$.
2. Define polar of a point with respect to a conic.
3. Define a conjugate hyperbola.
4. Write the general equation of a straight line in polar co-ordinates.
5. Write the polar equation of a circle when the pole is on its circumference and the diameter through it is taken as the initial line.
6. Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
7. Express $\tanh x$ in terms of e^x.
8. What is a singular matrix ?
9. State Cayley-Hamilton Theorem.
10. Define rank of a matrix.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*

Each question carries **2** *marks.*

11. If the tangents to $y^2 = 4\,ax$ at t_1 and t_2 meet at (h, k), prove that $\frac{h}{t_1\,t_2} = \frac{k}{t_1 + t_2} = a$.

Turn over

12. Find the condition that the lines $lx + my + n = 0$ and $l'x + m'x + n' = 0$ to be conjugate with respect to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

13. Find the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

14. Find the condition that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the circle $r = d\cos\theta$.

15. Find the equation of the tangent at a point on the conic $\frac{l}{r} = 1 + e\cos\theta$, whose vectorial angle is α.

16. Show that $\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$.

17. If $\tan \theta/2 = \tanh u/2$, show that $\sinh u = \tan\theta$.

18. Prove that $log\,(1-i) = \frac{1}{2}\log_2 + i\,(3\pi/4)$.

19. Prove that $\cos 4\theta = \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta$.

20. How can one reduce a matrix in Echelon form.

21. Find the characteristic polynomial of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

22. Show that the eigen values of a skew-Hermitian matrix are either zero or purely imaginary.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Prove that three normals can be drawn to a parabola from any point in its plane and that the sum of the ordinates of the feet of the normals is zero.

24. Find the locus of the poles of all tangents of the parabola $y^2 = 4\,ax$ with respect to the parabola $y^2 = 4\,bx$.

25. Derive the equation of a rectangular hyperbola referred to its asymptotes as axes of co-ordinates.

26. PSP^1 is a focal chord of a conic, focus S and SL is the semi latus rectum. Prove that $\frac{2}{SL} = \frac{1}{SP} + \frac{1}{SP^1}$.

27. Show that the sum of the infinite series $\frac{c\sin\theta}{1!} - \frac{c^3\sin 3\theta}{3!} + \frac{c^5\sin 5\theta}{5!} + \ldots\ldots$

$= \sin(c\sin\theta)\cosh(c\cos\theta)$.

28. Resolve $x^6 - 1$ into real factors.

29. By reducing to the normal form find the rank of $\begin{bmatrix} 3 & 1 & 2 & 5 \\ -1 & 4 & 1 & -1 \\ 1 & 9 & 4 & 3 \end{bmatrix}$.

30. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{bmatrix}$.

31. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries **15** *marks.*

32. (a) **Show that the locus of mid-points of chords of parabola which substands a right angle at the vertex is another parabola of half the latus rectum of the original parabola.**

(b) **Show that any tangent to a hyperbola cuts-off from the asymptotes a triangle of constant area.**

33. (a) **Derive the polar equation of a conic .**

(b) **Find the equation of the polar of any point (r_1, θ_1) with respect to the conic $\frac{l}{r} = 1 + e\cos\theta$.**

Turn over

34. (a) Sum the series $\cosh\alpha - \frac{1}{2}\cosh 2\alpha + \frac{1}{3}\cosh 3\alpha \ldots\ldots\ldots \infty$.

(b) Factorise $x^7 - 1$ into real factors.

35. (a) Using Cayley-Hamilton theorem show that $A^3 - 6A^2 + 11A - 6I = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence, find A^{-1}.

(b) Solve the system of equations :

$$5x + 3y + 3z = 48$$
$$2x + 6y - 3z = 18$$
$$8x - 3y + 2z = 21.$$

$(2 \times 15 = 30)$