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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2016

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and Model II and B.Sc. Computer Applications)

[2013 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State Cauchy's General Principle of Convergence for series.
2. Is the series $2+\frac{3}{2}+\frac{4}{3}+\ldots\ldots\ldots$ convergent ? Why ?
3. Define absolutely convergent series.
4. Define discontinuity of the first kind.
5. Define partition of $[a, b]$ and refinement of a partion.
6. Define intermediate value property of a function f on $[a, b]$.
7. Let $f(x) = 1$ for all x in $[0, 1]$. Is f integrable on $[0, 1]$? Why or why not ?
8. State the fundamental theorem of calculus.
9. Show that the series $\sum \frac{\cos n\theta}{2^n}$ converges uniformly for all real values of θ.
10. State Abel's test.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
2 *marks each.*

11. Prove or disprove : If $u_n \to 0$ then $\sum u_n$ is convergent.
12. Test the convergence of the series :

$$\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\ldots\ldots$$

Turn over

13. State D'Alembert's ratio test.

14. Prove that every absolutely convergent series is convergent.

15. Define removable discontinuity. Illustrate it with an example.

16. If a function f is continuous on $[a, b]$ and $f(x) \in [a, b]$ for every x in $[a, b]$, prove that there exists a point c in $[a, b]$ such that $f(c) = c$.

17. Prove that $\sin x$ is uniformly continuous on $[0, \infty]$.

18. Explain upper integral and lower integral of a bounded function f on $[a, b]$.

19. Let f be a bounded function on $[a, b]$. For any partitions P_1, P_2, prove that $L(P_1, f) \le U(P_2, f)$.

20. Prove or disprove : If $|f|$ is integrable on $[a, b]$, then f is integrable on $[a, b]$.

21. Explain pointwise convergence of a sequence of functions.

22. Show that the sequence $\{f_n\}$ where $f_n(x) = \dfrac{x}{x+n}$ is uniformly convergent on $[0, k]$ for any $k > 0$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*

4 marks each.

23. Discuss the convergence of the positive term geometric series $1 + r + r^2 + r^3 + \ldots$

24. State and prove Cauchy's root test.

25. State Leibnitz test. Use it to show that the series $\dfrac{1}{1^p} - \dfrac{1}{2^p} + \dfrac{1}{3^p} - \dfrac{1}{4^p} + \cdots$ converges for $p > 0$.

26. Prove that a continuous function on a closed interval I is bounded on I.

27. State and prove Intermediate value theorem.

28. If f is integrable on $[a, b]$, prove that f^2 is integrable on $[a, b]$.

29. Compute $\int_{-1}^{1} f\, dx$ where $f(x) = |x|$.

30. State and prove Cauchy criterion for uniform convergence of a sequence of functions.

31. Prove that the sequence $\{f_n\}$ where $f_n(x) = \dfrac{x}{1+nx^2}$, x real converges uniformly on any closed interval.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
15 *marks each.*

32. (a) State and prove Gauss's test.

(b) Test for convergence the series $\sum \frac{1^2\, 3^2 \,....\, (2n-1)^2}{2^2\, 4^2 \,....\, (2n)^2} x^{n-1}$, $x > 0$.

33. (a) Prove that a function f defined on an interval I is continuous at a point c in I if and only if for every sequence $\{c_n\}$ in I converging to c we have $\lim_{n \to \infty} f(c_n) = f(c)$.

(b) Give an example of a function on R which is discontinuous at every point. Justify your example.

34. (a) Prove that a bounded function f is integrable on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

(b) If f is a non-negative continuous function on $[a, b]$ and $\int_a^b f\, dx = 0$, prove that $f(x) = 0$ for all x in $[a, b]$.

35. (a) Show that the sequence $\{f_n\}$ where $f_n(x) = \tan^{-1} nx$ is uniformly convergent in any interval $[a, b]$, $a > 0$ but is only pointwise convergent in $[0, b]$.

(b) State and prove Weierstrass M-test.

(c) Test for uniform convergence the series $\sum \frac{\sin n\theta}{n^2}$ for real values of θ.

$(2 \times 15 = 30)$