E 7513 (Pages : 3) Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014

Sixth Semester

Core Course—DISCRETE MATHEMATICS

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions has weight* **1.**

I. 1. Give an example of a graph which is both complete and complete bipartite.

2. Define subgraph of a graph G.

3. If $d_G^{(V)} = K$ then find $d_{\bar{G}}^{(V)}$?

4. Define adjacency matrix of a graph G.

II. 5. State Cayley's theorem on spanning trees.

6. Define n-connected graph G.

7. Define Euler tour.

8. Give an example of a Hamiltonian graph.

III. 9. Define perfect matching.

10. State Hall's Marriage theorem.

11. Give an example of a simple graph G such that L(G) is Euler but G is not.

12. Define "autokey".

IV. 13. Is the sequence 3, 13, 20, 37, 81 superincreasing ?

14. Give an example of a lattice.

15. Define sub-lattice.

16. Define a Chain.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight of 1.

17. In any graph G, prove that there is an even number of odd vertices.

18. List all the self-complementary graphs with 4 *or* 5 vertices.

19. Write a note on Konigsberge bridge problem.

20. Which of the complete graphs K_n's are Euler ? Justify.

21. State two characterisation of trees.

22. Decrypt the message

    RXQTGUHOZTKGHFJKTMMTG, which was produced using the linear cipher $C \equiv 3p + 7 \pmod{26}$.

23. Prove that a chain is a distributive lattice.

24. Prove that a finite lattice has least and greatest elements.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25. Prove that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G.

26. Prove that a graph G is connected if and only if it has a spanning tree.

27. Prove that a connected graph G has an Euler trial if and only if it has at most two odd vertices.

28. Prove that a 2-regular graph G has a perfect matching if and only if each component of G is an even cycle.

29. Find the unique solution of the super increasing knapsack problem

    $118 = 4x_1 + 5x_2 + 10x_3 + 20x_4 + 41x_5 + 99x_6$.

30. Prove that duel of a complemented lattice is complemented.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has a weight of 4.

31. Define bipartite graph. Let G be a non-empty graph with atleast two vertices. Then prove that G is bipartite if and only if it has no odd cycles.

32. If G is a simple graph with n vertices where $n \geq 3$, and the degree $d(V) \geq n/2$ for every vertex V of G, then prove that G is Hamiltonian.

33. Define lattice as a poset and as an algebra. Show that these two definitions are equivalent.

$(2 \times 4 = 8)$