

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2016****Fourth Semester**

Complementary Course—Operations Research

**NON-LINEAR PROGRAMMING**

(For B.Sc. Mathematics Model II)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

**Part A**

*Answer all the questions.  
Each question carries 1 mark.*

1. Define a mixed integer programming problem.
2. Define a 0—1 integer programming problem.
3. Define a convex set.
4. How does the optimal solution of an integer programming problem compared with that of the linear programming problem.
5. When a subproblem is said to be fathomed in branch and bound method.
6. What is meant by quadratic programming ?
7. Is it correct to say that in the quadratic programming the objective equation and the constraints both should be quadratic.
8. Define convex programming.
9. Define saddle point.
10. Give an example of a separable function.

(10 × 1 = 10)

**Part B**

*Answer any eight questions.  
Each question carries 2 marks.*

11. Write the steps involved in Gomory's all integer algorithm.
12. Write the general form of an ILP and the related linear programming problem.
13. "Addition of a cut makes the previous non-integer optimal solution infeasible". Explain.
14. Discuss the advantages of branch and bound method.

Turn over

15. Explain the merits and demerits of 'rounding off' a continuous optimal solution to a LP problem to obtain an integer solution.
16. If  $F(X, Y)$  has a saddle point  $(X_0, Y_0)$  for every  $Y \geq 0$ , then show that  $G(X_0) \leq 0, Y_0^T G(X_0) = 0$ , where  $G(X) = [q_1(x) \ q_2(x) \dots \ q_m(x)]^T$  and  $F(X, Y) = f(x) + y' G(x)$ .
17. Minimize  $f = x_1^2 + x_2^2$  subject to  $g = (x_1 - 1)^3 - x_1^2 \geq 0$ .
18. Write the Kuhn-Tucker conditions for Minimize  $f = (x_1 + 1)/(x_2 - 2)$  over the region  $0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1$ .
19. How does a quadratic programming problem differ from a L.P.P.
20. Write the mathematical formulation of a general non-linear programming problem.
21. Define a separable Programming Problem.
22. Give an example of a non-linear programming problem which is not convex.

(8 × 2 = 16)

**Part C**

*Answer any six questions.  
Each question carries 4 marks.*

23. Prove that an optimal solution of

$$\text{Minimise } f(X) = CX$$

$$\text{subject to } X \in T_F$$

is an optimal solution of

$$\text{Minimize } f(X) = CX$$

$$\text{subject to } X \in [T_F]$$

24. Solve by Cutting Plane method minimise  $x_1 + x_2$  subject to  $7x_1 - 6x_2 \leq 5, 6x_1 + 3x_2 \geq 7, -3x_1 + 86x_2 \leq 6, x_1, x_2$  non-negative integers.
25. Solve by the branch and bound method maximise  $11x_1 + 21x_2$  subject to  $4x_1 + 7x_2 + x_3 = 13, x_1, x_2, x_3$  non-negative integers.



26. Solve the knapsack problem with the following data :

| Object | Weight | Value |
|--------|--------|-------|
| $j$    | $w_j$  | $v_j$ |
| 1      | 2      | 10    |
| 2      | 2      | 14    |
| 3      | 3      | 18    |
| 4      | 6      | 48    |
| 5      | 8      | 80    |

Knapsack Capacity  $W = 12$ .

27. If  $X_0$  is a solution of the convex programming problem, Minimise  $f(X)$ ,  $X \in E_n$  subject to  $g_i(X) \leq 0, i = 1, 2, \dots, m$ , where  $f(X)$ ,  $g_i(X)$  are convex functions, with the set of points of  $X$ , such that  $G(X) < 0$  is non empty. Prove that there exists a non-negative vector  $Y_0$  in  $E_m$  such that  $(X_0, Y_0)$  is a saddle point of  $F(X, Y) = f(X) + Y'G(X)$ .
28. Solve the problem graphically maximize  $(x_1 - 4)^2 + (x_2 - 4)^2$  subject to the constraints,  
 $x_1 + x_2 \leq 6, x_1 - x_2 \leq 1, 2x_1 + x_2 \geq 6, \frac{1}{2}x_1 - x_2 \geq -4, x_1 \geq 0, x_2 \geq 0$ .
29. Solve by K—T conditions and verify geometrically, the problem minimise  $x_1$  subject to  
 $(x_1 - 4)^2 + x_2^2 \leq 16, (x_1 - 3)^2 + (x_2 - 2)^2 = 13$ .
30. Maximise  $2x_1 - x_1^2 + x_2$  subject to  $2x_1 + 3x_2 \leq 6, 2x_1 + x_2 \leq 4, x_1 \geq 0, x_2 \geq 0$ .
31. Show that the K—T conditions fails to give max  $x_1$  subject to  $(1 - x_1)^3 - x_2 \geq 0, x_1 \geq 0, x_2 \geq 0$ .

(6 × 4 = 24)

### Part D

Answer any **two** questions.  
 Each question carries 15 marks.

32. Maximise  $2x_1 + 5x_2$ , subject to  $0 \leq x_1 \leq 8, 0 \leq x_2 \leq 8$  and either  $4 - x_1 \geq 0$  or  $4 - x_2 \geq 0$ .
33. Express the following conditions as simultaneous constraints using 0—1 variables
- (i) Either  $x_1 + 2x_2 \leq 4$  or  $2x_1 + 3x_2 \geq 12$ .

Turn over

(ii) If  $x_3 \leq 4$  then  $x_4 \geq 5$ , otherwise  $x_4 \leq 2$ .

(iii)  $x_5 = 1$  or 3 or 5 only.

(iv) At least two of the following constraints are satisfied  $x_6 + x_7 \leq 3$ ,  $x_6 \leq 2$ ,  $x_7 \leq 4$ ,  $x_6 + x_7 \geq 5$ .

34. Solve by the method of quadratic programming

$$\text{Minimize } -x_1 - x_2 - x_3 + \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

$$\text{subject to } x_1 + x_2 + x_3 - 1 \leq 0,$$

$$4x_1 + 2x_2 - \frac{7}{3} \leq 0$$

$$x_1, x_2, x_3 \geq 0.$$

35. Solve using separable programming maximize  $9 - (x_1 - 3)^2 - (x_2 - 2)^2$  subject to

$$4x_1 + x_2^2 \leq 16, x_1 \geq 0, x_2 \geq 0.$$

(2 × 15 = 30)