

E 7511

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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and II and B.Sc. Computer Applications)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type)

Answer all questions.

Each bunch of 4 questions has weight 1.

- I. 1 Show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$ is not convergent.
- 2 State D'Alembert's ratio test.
- 3 State Raabe's test for convergence of a series.
- 4 What is an alternating series ?
- II. 5 What do you mean by absolute convergence of a series ?
- 6 Is the series $1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \dots$ absolutely convergent.
- 7 What do you mean by jump discontinuity ?
- 8 Define uniform continuity.
- III. 9 Show that the function defined by $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$ has a removable discontinuity at the origin.
- 10 At which Point the function $f(x) = \frac{1}{1+|x|}$ for real x , attain its supremum.
- 11 Define the upper sum of a bounded function of defined on a closed bounded interval.
- 12 Define Riemann integral of a bounded function.

Turn over

IV. 13 State fundamental theorem of calculus.

14 State Cauchy's criterion for uniform convergence.

15 State Dirichelet's test for uniform convergence.

16 Test for uniform convergence the series $\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \dots, -\frac{1}{2} \leq x \leq \frac{1}{2}$.

(4 × 1 = 4)

Part B

Answer any five questions.

Each question has weight 1.

17 Prove that the alternating series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots$ is convergent.

18 Show that the series $\sum \frac{1-n}{1+2n}$ diverges.

19 Investigate the behaviour of $\sum a_n$ if $a_n = (\sqrt[n]{n} - 1)^n$.

20 Show that the function defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is continuous at $x = 0$.

21 Show that a function which is uniformly continuous on an interval is continuous on that interval.

22 Define Riemann sum of a bounded function f over $[a, b]$ relative to a partition P .

23 Show by an example that integrability of $|f|$ need not imply the integrability of f .

24 Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{nx}{1+n^2 x^2}$ is not uniformly convergent on any interval containing zero.

(5 × 1 = 5)

Part C

*Answer any four questions.
Each question has weight 2.*

- 25 Test the convergence of the series $\sum \frac{n^2 - 1}{n^2 + 1} x^n$.
- 26 Show that the series $\frac{1}{1p} - \frac{1}{2p} + \frac{1}{3p} \dots$ Converges for $p > 0$.
- 27 Show that a function which is continuous on a closed interval $[a, b]$, assumes every value between its bounds.
- 28 Show that a constant function k is integrable on $[a, b]$ and $\int_a^b k dx = k(b - a)$.
- 29 If a function f is bounded and integrable on each of the intervals $[a, b]$, $[c, b]$ and $[a, c]$, where c is a point in $[a, b]$, then show that $\int_a^b f dx = \int_a^c f dx + \int_c^b f dx$.
- 30 State and prove Weierstrass μ -test.

(4 × 2 = 8)

Part D

*Answer any two questions.
Each question has weight 4.*

- 31 State and prove Cauchy's general principle of convergence of a series. Use this test to show that the series $\sum \frac{1}{n}$ does not converge.
- 32 Show that if a function f is continuous on a closed interval $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs then there exists atleast one point $\alpha \in [a, b]$ such that $f(\alpha) = 0$.
- 33 State and prove Fundamental theorem of calculus.

(2 × 4 = 8)