



QP CODE: 22103519



22103519

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,  
NOVEMBER 2022  
Fifth Semester**

**CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS**

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science &  
B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

A4D9D311

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define absolute value function.
2. If  $t > 0$  prove that there exist an  $n_t \in \mathbb{N}$  such that  $0 < \frac{1}{n_t} < t$
3. Is any intervals are finite set? Justify.
4. Is the number 1.31311311131111... rational? Give proper reasoning.
5. Show that  $\lim(\frac{1}{3^n}) = 0$ .
6. Prove that (n) is divergent.
7. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
8. Prove that  $(1+(-1)^n)$  is not Cauchy.
9. Define properly divergent sequences. Give an example.
10. State Raabe's Test
11. State any two conditions for the convergence of the series  $\sum x_n y_n$ .
12. True or False:" The set  $A = \{1, 2\}$  has no cluster points". Give justifications.

(10×2=20)





### Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Suppose  $S, T$  are sets such that  $S \subset T$  then prove that
  - (a.) If  $T$  is finite, then  $S$  is finite.
  - (b) If  $S$  is infinite, then  $T$  is infinite
14. State and prove the density theorem on rational numbers? Can the same argument be said about irrationals, verify?
15. Prove that the  $m$ -tail of a sequence converges if and only if the sequence converges.
16. Let  $X = (x_n)$  is a sequence of real numbers and  $Y = (y_n)$  is a sequences of non-zero real numbers that converges to  $x$  and  $y \neq 0$  respectively. Prove that the sequences  $X/Y$  converges to  $x/y$ .
17. Let  $x_1 = 2$  and  $x_{n+1} = 2 + \frac{1}{x_n}$ . Prove that  $\lim(x_n) = 1 + \sqrt{2}$ .
18. If  $\sum a_n$  is convergent, then prove that any series obtained from it by grouping terms is also convergent to the same value.
19. If  $\sum a_n$  is a convergent series of real numbers then is it necessary that  $\sum \frac{\sqrt{a_n}}{n}$  is convergent?
20. Evaluate the one-sided limits of the function  $h(x) = \frac{1}{(e^{\frac{1}{x}} + 1)}$  at  $x = 0$ .
21. Let  $A \subseteq \mathbb{R}$ ,  $f, g : A \rightarrow \mathbb{R}$ ,  $c \in \mathbb{R}$  be a cluster point of  $A$ . If  $f(x) \leq g(x)$  for all  $x \in A, x \neq c$ , Then prove the following
  - (a) If  $\lim_{x \rightarrow c} f = \infty$ , then  $\lim_{x \rightarrow c} g = \infty$ .
  - (b) If  $\lim_{x \rightarrow c} g = -\infty$ , then  $\lim_{x \rightarrow c} f = -\infty$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Prove that the set of all real numbers is a complete ordered field?
23. (a) Prove that for any real number  $a > 0$ , there exists a sequence  $(s_n)$  of real numbers that converges to  $\sqrt{a}$ .  
 (b) What is Euler number. Prove that Euler number lies between 2 and 3.





24. (a) State and prove the Limit Comparison Test for the convergence of series.  
(b) Discuss the convergence of

- $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$
- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$

25. (a) If  $A \subseteq \mathcal{R}$  and  $f : A \rightarrow \mathcal{R}$  has a limit at  $c \in \mathcal{R}$ , then prove that  $f$  is bounded on some neighborhood of  $c$ .  
(b) Show that  $\lim_{x \rightarrow 0} \frac{1}{x}$  does not exist in  $\mathcal{R}$ .  
(c) Evaluate the limit"  $\lim_{x \rightarrow 0} \cos x$ ".

(2×15=30)

