

QP CODE: 24020945



Reg No :

Name :

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, APRIL 2024

Fourth Semester

**Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND
LAPLACE TRANSFORMS**

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc
Mathematics Model II Computer Science)

2017 Admission Onwards

C8994E19

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give a vector equation for the line L through $P_0(x_0, y_0, z_0)$ parallel to \mathbf{v} .
2. Write the vector equation and component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
3. Define the circle of curvature and centre of curvature for plane curves.
4. Find the acceleration for the position vector $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j}$ at $t = 0$.
5. Find the potential function f for the field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$.
6. Find the divergence of the vector field $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$.
7. Prove: If $a \equiv b \pmod{n}$ and $m|n$, then $a \equiv b \pmod{m}$.
8. Verify that $5^{38} \equiv 4 \pmod{11}$ using Fermat's theorem.
9. Show that $18! \equiv -1 \pmod{437}$.
10. Define Laplace transform of a function and hence prove that $\mathcal{L}(e^{at}) = \frac{1}{s-a}$.
11. State first shifting theorem for Laplace Transform.
12. Evaluate $\mathcal{L}(\sin^2 \omega t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Graph the vector function $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$ giving the complete details.





14. The cylinder $f(x, y, z) = x^2 + y^2 - 2 = 0$ and the plane $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E . Find parametric equations for the line tangent to E at the point $P_0(1, 1, 3)$.
15. a) Explain component test for conservative field.
b) Give example of a field which is not conservative.
16. Using spherical co-ordinate system, find the surface area of a sphere of radius a .
17. Find the divergence and curl of $F = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at $(1, 2, -1)$.
18. Let a and b are integers that are not divisible by the prime p , then if $a^p \equiv b^p \pmod{p}$ prove that $a^p \equiv b^p \pmod{p^2}$.
19. Let n be a composite square-free integer, say, $n = p_1 p_2 \dots p_r$, where the p_i are distinct primes.
If $p_i - 1 \mid (n - 1)$ for $i = 1, 2, \dots, r$, then prove that n is an absolute pseudoprime.
20. Find $\mathcal{L}^{-1} \left\{ \frac{s^2 + 2s + 5}{(s-1)(s-2)(s-3)} \right\}$.
21. Using convolution theorem, solve $y'' + 5y' + 4y = 2e^{-2t}$, $y(0) = 0$, $y'(0) = 0$.
(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

- 22.
1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the curve $x^2 + y^2 = 4$ at the point $(\sqrt{2}, \sqrt{2})$.
 2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0 , and what are the rates of change in these directions?
23. State and verify Green's Theorem (any one form) for the vector field $F(x, y) = (y - x)i + yj$ and the region bounded by the unit circle $C: r(t) = (\sin t)i + (\cos t)j, 0 \leq t \leq 2\pi$.
- 24.
1. Prove that the Euler phi-function is a multiplicative function.
 2. Prove: For $n > 2$, $\phi(n)$ is an even integer.
- 25.
1. Solve $y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$ using Laplace Transform.
 2. Solve the Volterra integral equation
$$y(t) - \int_0^t (1 + \tau)y(t - \tau)d\tau = 1 - \sinh t.$$

(2×15=30)

