



22102799

QP CODE: 22102799

Reg No :

Name :

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, AUGUST 2022**Fourth Semester****Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS**

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2020 Admission Only

40FC1795

Time: 3 Hours

Max. Marks : 80

Part A*Answer any **ten** questions.**Each question carries **2** marks.*

1. Define an **arc length parameter** for a smooth space curve.
2. Define the curvature of a smooth plane curve. Give a formula for calculating curvature of a smooth plane curve $\mathbf{r}(t)$.
3. Define the **tangent plane** and the **normal line** at a point on a smooth surface in space.
4. Find the acceleration for the position vector $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j}$ at $t = 0$.
5. Find the potential function f for the field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$.
6. Define flux of a continuous vector field \mathbf{F} across an oriented surface S in the positive direction in terms of double integral.
7. Check whether the integer 1729 is an *absolute pseudoprime* or not.
8. Define quadratic congruence with example.
9. Prove $\phi(n) = n - 1$ if and only if n is prime.
10. Define Laplace transform of a function and hence prove that $\mathcal{L}(e^{at}) = \frac{1}{s-a}$.
11. Find $\mathcal{L}^{-1} \left\{ \frac{\sqrt{8}}{(s+\sqrt{2})^3} \right\}$.





12. Evaluate $\mathcal{L}(\sin^2 \omega t)$.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Give the vector equation and component equation for the plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$. Also find an equation for the plane through $A(0, 0, 1)$, $B(2, 0, 0)$ and $C(0, 3, 0)$.
14. Define derivative for a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$. If \mathbf{r} is the position vector of a particle moving along a smooth curve in space, then define the particle's velocity vector, direction of motion, speed and acceleration vector.
15. Find the flux of the field $F = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ across the triangle with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$.
16. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.
17. Prove that for a scalar function $f(x, y, z)$, $\text{curl}(\text{grad } f) = 0$.
18. Prove: If $ca \equiv cb \pmod{n}$, then $a \equiv b \pmod{\frac{n}{d}}$, where $d = \text{gcd}(c, n)$.
19. Derive the congruence: $a^9 \equiv a \pmod{30}$ for all a .
20. Using convolution theorem, solve $y'' + 5y' + 4y = 2e^{-2t}$, $y(0) = 0$, $y'(0) = 0$.
21. Solve $y(t) + 2e^t \int_0^t e^{-\tau} y(\tau) d\tau = t e^t$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22.

1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the curve $x^2 - y = 1$ at the point $(\sqrt{2}, 1)$.
2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0 , and what are the rates of change in these directions?





23. a) State Stoke's Theorem.
b) Find the Circulation of the field $F = (x^2 - y)i + 4zj + x^2k$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above.
- 24.
1. State and prove Fermat's theorem.
 2. Prove: If p is a prime, then $a^p \equiv a \pmod{p}$ for any integer a .
- 25.
1. Let $f(t)$, $f'(t)$ be continuous and satisfy the growth restriction for all $t \geq 0$.
Let $f''(t)$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Prove that the Laplace transform of $f''(t)$ satisfies
 $\mathcal{L}(f'') = s^2 \mathcal{L}(f) - sf(0) - f'(0)$.
 2. Solve the Initial value problem $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = -3$.

(2×15=30)

