

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012**Fifth Semester****Core Course—DIFFERENTIAL EQUATIONS**

(Common for Model I and Model-II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type Questions)*Answer all questions.**Each bunch of four questions has weight 1.*

- I. 1. Write the condition for $M(x, y)dx + N(x, y)dy = 0$ to be exact.
2. Find an integrating factor of the equation $\frac{dx}{dy} + \frac{2}{y}x = y^2$.
3. Define Bernoulli differential equation.
4. Find the differential equation of the orthogonal trajectories of the family of curves $y = cx^2$.
- II. 5. Find the general solution of $\frac{d^2y}{dx^2} + y = 0$.
6. Write the UC set of $x^2 e^x$.
7. Write the transformation which reduce a Cauchy-Euler equation to a linear differential equation with constant coefficients.
8. Find the Wronskian $W(x^2, x^3)$.
- III. 9. Define singular point of a second order homogeneous linear differential equation.
10. The equation $x^2(x-4)^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + (x-4)y = 0$ has an irregular singular point at $x = \underline{\hspace{2cm}}$.
11. If $p > 0$ is not an integer, then the general solution of the Bessel equation of order p is $\underline{\hspace{2cm}}$.
12. Find $(D^2 + 1)(D + 2)t^3$, where $D \equiv \frac{d}{dt}$.

Turn over

- IV. 13. The direction cosines of the normal to the surface $z = f(x, y)$ at the point (x, y, z) are _____.
14. Write the second order differential equation $\frac{d^2x}{dt^2} = f\left(t, x, \frac{dx}{dt}\right)$ as a system of two first order equations.
15. Eliminate the constants a and b from $z = (x + a)(x + b)$.
16. What is Lagrange's partial differential equation ?

(4 × 1 = 4)

Part B (Short Answer Type Questions)*Answer any five questions.**Each question has weight 1.*

17. Solve : $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$.
18. Find an integrating factor of the equation $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$.
19. Find a particular integral of the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$.
20. Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0$, given that $x > 0$.
21. Locate and classify the singular points of $(x^2 - 3x) \frac{d^2y}{dx^2} + (x + 2) \frac{dy}{dx} + y = 0$.
22. Find the indivial equation of $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - 1)y = 0$.
23. Find the partial differential equation of the family of curves $x^2 + y^2 = (z - c)^2 \tan^2 \alpha$, where both constants c and α are arbitrary.
24. Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** questions.
Each question has weight 2.

25. Verify the exactness and solve the equation $(y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0$.
26. Solve: $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$.
27. Solve: $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 12y = 0$, $y(0) = 3$, $y'(0) = 5$.
28. Apply the method of variation of parameters to solve $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = \frac{e^{-x}}{x^3}$.
29. Find the integral curves of the sets of equations $\frac{a dx}{(b-c)yz} = \frac{b dy}{(c-a)zx} = \frac{c dz}{(a-b)xy}$.
30. Find the general integral of $y^2 p - xy q = x(z - 2y)$.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** questions.
Each question has weight 4.

31. (a) If $M(x, y) dx + N(x, y) dy = 0$ is a homogeneous equation, prove that the change of variables $y = vx$ transform the equation into a separable equation in the variables v and x .
- (b) Solve the equation $(x^2 - 3y^2) dx + 2xy dy = 0$.
32. Solve the initial-value problem $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - 3y = 2e^x - 10 \sin x$, $y(0) = 2$, $y'(0) = 4$.
33. Solve the system:

$$2 \frac{dx}{dt} - 2 \frac{dy}{dt} - 3x = t.$$

$$2 \frac{dx}{dt} + 2 \frac{dy}{dt} + 3x + 8y = 2.$$

(2 × 4 = 8)