

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2017**Second Semester****Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES**

(Common for B.Sc. Mathematics Model I, Model II and B.Sc. Computer Application)

(2018 Admission onwards)

Time : Three Hours

Maximum Marks : 80

Part A*Answer all questions.**Each question carries 1 mark.*

1. What is the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$?
2. How many normals can be drawn from a given point to a parabola?
3. Define director circle of an ellipse.
4. What is the condition for the line $lx + my + n = 0$ to touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?
5. What is a rectangular hyperbola?
6. What is the equation of the tangent at a point θ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?
7. Prove that $\cosh 2\theta = 1 + 2 \sinh^2 \theta$.
8. Separate $\cos(\alpha + i\beta)$ into real imaginary parts.
9. Define rank of a matrix.
10. State Cayley-Hamilton theorem.

(10 × 1 = 10)

Part B*Answer any eight questions.**Each question carries 2 marks.*

11. Define a conjugate hyperbola.
12. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
13. Show that if $\alpha - \beta$ is a constant, the chord joining the points α and β on an ellipse touches a fixed ellipse.

Turn over

14. What is the pole of the line $lx + my + n = 0$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
15. Show that in a parabola if the normal at P meets the axis in G, then $SG = SP$, where S is the focus.
16. If the normal at the point f_1 on a parabola $y^2 = 4ax$ meet it again at t_2 . Prove that $t_2 = -t_1 - \frac{2}{t_1}$.
17. Find the point of intersection of the tangents at θ and ϕ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
18. If the normal at a point P on the ellipse meets the major axis at G, then show that $SG = C.SP$.
19. Find the condition in order that the line $\frac{l}{r} = A \cos \theta + B \sin \theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e \cos \theta$.
20. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}$.
21. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.
22. If $\tan \frac{\theta}{2} = \tanh \frac{u}{2}$, show that $u = \log \tan \left[\frac{\pi}{4} + \frac{\theta}{2} \right]$.

(8 × 2 = 16)

Part C

*Answer any six questions.
Each question carries 4 marks.*

23. Separate $\log(\alpha + i\beta)$ into real and imaginary parts.
24. If $\sin(\alpha + i\beta) = \cos \theta + i \sin \theta$, prove that $\cos^2 \alpha = \sin^2 \beta$.
25. Show that $\frac{c \sin \theta}{1!} + \frac{c^3 \sin 3\theta}{3!} + \frac{c^5 \sin 5\theta}{5!} + \dots$
 $= \sin(c \sin \theta) \cosh(c \cos \theta)$.
26. Sum to infinity $\sinh \alpha - \frac{1}{2} \sinh 2\alpha + \frac{1}{3} \sinh 3\alpha - \dots - \infty$.
27. Find the asymptotes of the conic $\frac{l}{r} = 1 + e \cos \theta$.

28. Obtain the polar equation of a circle.
29. Prove that the chords of a rectangular hyperbola which subtend a right angle at a focus touch a fixed parabola.
30. Using Cayle-Hamilton theorem, show that $A^3 - 6A^2 + 11A - 6I = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence find A^{-1} .
31. Test for consistency and solve the system of equations

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6.$$

(6 × 4 = 24)

Part D

*Answer any two questions.
Each question carries 15 marks.*

32. (i) Prove that a circle will cut a parabola in four points and the algebraic sum of the ordinates of the four points is zero.
- (ii) If P and D are extremities of conjugate diameters of the ellipse. Show that the locus of the point of intersection of the tangents at P and D is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.
33. (i) Find the locus of the feet of the perpendiculars from the centre on normals to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
- (ii) Prove that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is constant.
34. (i) Factorize $x^8 + 1$ into real factors.
- (ii) Sum the series $1 + \frac{1}{2} \cos^2 \theta - \frac{1}{2.4} \cos 4\theta + \frac{1.3}{2.4.6} \cos 6\theta \dots$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
35. (i) Solve by Cramers rule

$$x + y + z = 5$$

$$x - 2y - 3z = -1$$

$$2x + y - z = 3$$

- (ii) Determine the eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(2 × 15 = 30)