E 1359 (Pages : 3) Reg. No................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Fourth Semester

Complementary Course—Mathematics

FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

(For the programme B.Sc. Physics / Chemistry / Petrochemicals / Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013 admissions]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 1 mark.

1. Find the fundamental period of $\sin(2\pi x)$.
2. Write the Bessel function of first kind of order v.
3. Write a parametric equation of the sphere $x^2 + y^2 + z^2 = a^2$.
4. Write the partial differential equation representing $x^2 + y^2 + (z-c)^2 = a^2$, where a and c are arbitrary constants.
5. Write the Lagrange's partial differential eequation.
6. Round off 1.6583 to four significant figures.
7. If $u = 3V^7 - 6V$. Find the percentage error in u at $v = 1$, if the error in V is 0.05.
8. Find the order of the subgroup of Z_4 generated by 3.
9. How many homomorphisms are there of Z onto Z.
10. Give a basis for $Q(\sqrt{2})$ over Q.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Sketch the graph of :

$$f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$$

Turn over

12. Solve the equation $y' - y = 0$ by the method of power series.

13. Eliminate the constants a and b from the equation $ax^2 + by^2 + z^2 = 1$.

14. Show that the direction cosines of the tangent of the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

15. Eliminate the arbitrary function from the equation $z = xy + f\left(x^2 + y^2\right)$.

16. Evaluate $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors.

17. Find a real root of the equation $f(x) = x^3 - x + 1 = 0$ by bisection method.

18. Evaluate $f(1)$ using Taylor's series for $f(x) = x^3 - 3x^2 + 5x - 10$.

19. Evaluate $\sqrt[3]{24}$ (correct to 4 decimal places) by Newton's iteration method.

20. Prove that every cyclic group is abelian.

21. Prove that intersection of subspaces of a vector space V over F is again a subspace of V.

22. Consider the matrix ring $M_2(Z_2)$. Find the order of the ring and list all units in the ring.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*

Each question carries 4 marks.

23. Find the Fourier series for the function $f(x) = \begin{cases} -1, \text{if } 0 < x < \pi/2 \\ 0, \text{if } \pi/2 < x < 2\pi \end{cases}$.

24. Find a solution of $\left(a^2 - x^2\right)y'' - 2xy' + 12y = 0, a \neq 0$.

25. Find the integral curves of :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$

26. Find the general integral of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.

27. Find the Fourier cosin series as well as the Fourier sine series of $p(x) = \pi - x, 0 < x < \pi$.

28. Use method of false position to obtain a root correct to 3-decimal places of $x^3 - x - 1 = 0$.

29. The Maclaurin expansion of sin x is given by $\sin x = x - \frac{x^3}{3!} \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$, where x is in radians. Use the series to compute the value of sin 25° with an accuracy of 0.001.

30. Show that a subgroup of a cyclic group is cyclic.

31. Show that a group homomorphism $\phi : G \to G'$ is a one-to-one map if and only if $\ker \phi = \{e\}$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Solve by the method of power series $y'' + qy = 0$.

(b) Find the Fourier series for the function :

$$f(x) = \begin{cases} 1, & -1 < x < 0, p = 2L = 2 \\ -1, & 0 < x < 1 \end{cases}$$

(c) Find the Fourier series for :

$$f(x) = \begin{cases} x & \text{if } 0 < x < \pi \\ \pi - x, & \text{if } \pi < x < 2\pi \end{cases}$$

33. (a) Solve the equation :

$$\frac{a\,dx}{(b-c)yz} = \frac{b\,dy}{(c-a)2x} = \frac{c\,dz}{(a-b)xy}.$$

(b) Find the general integral of $y^2 p - xyq = x(z - 2y)$.

(c) Find the partial differential equation of all spheres whose centres lie on the z-axis.

34. (a) Use Newton-Raphson method to obtain a root of $x^3 + 3x^2 - 3 = 0$ correct to three decimal places.

(b) Use quotient difference method to obtain the approximate roots of $x^3 - x^2 - 2x + 1 = 0$.

35. (a) Show that the set of all complex numbers with usual addition and multiplication is a field.

(b) Give the multiplication table for the cyclic subgroup of S_5 generated by :

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}.$$

Is this group isomorphic to S_3.

(2 × 15 = 30)