E 6280 (Pages : 3) Reg. No..............................

Name..............................

# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2013

## Fifth Semester

Core Course—DIFFERENTIAL EQUATIONS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

### Part A (Objective Type Questions)

*Answer* **all** *the questions.*
*Each bunch of* **four** *questions has weight* 1.

I. 1 Test whether the following equation is exact $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0$.

2 Which substitution transforms the equations of the form $\frac{dy}{dx}=g\left(y/x\right)$ into a separable equation.

3 What is the general form of Bernoulli's differential equation.

4 Find the differential equation of the family of curves $2y-x+c=0$.

II. 5 Are the solutions $e^x$ and $xe^x$ of $y''-y'+y=0$ linearly independent.

6 Write the Euler-Cauchy differential equation.

7 How many arbitrary constants does a general solution of a non-homogeneous linear equation involve.

8 For the equation $y''+py'+qy=0$ where $p$ and $q$ are constants, if the roots of the indicial equations are of the form $a\pm ib$, what will be the form of general solution.

III. 9 Define a power series.

10 Find the radius of convergence of $1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots$

11 Write the singular points of $\left(1-x^2\right)y''-2x\,y'+p\,(p+1)\,y=0$.

12 Write the general form of Bessel's equation.

Turn over

IV. 13 Write a parametric equation of the surface $x^2 + y^2 + z^2 = a^2$.

14 Eliminate $a$ and $c$ from $x^2 + y^2 + (2-c)^2 = a^2$.

15 What is Lagrange's partial differential equation.

16 Is the equation $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = z^2 + x^2$ linear.

(4 × 1 = 4)

**Part B (Short Answer Type Questions)**

*Answer any* **five** *questions.*
*Each question has weight* 1.

17 Solve $y' + 3x^2y^2 = 0$.

18 Find an integrating factor of $2\cosh x \cos y \, dx = z \sinh x \sin y \, dy$.

19 Find the general solution of $y'' + 9y' + 20\,y = 0$.

20 Solve $x^2\, y'' - 2.5\, xy' - 2.0\, y = 0$.

21 Find the indicial equation and the roots

$$x^3\, y'' + (\cos 2x - 1)\, y' + 2xy = 0$$

$$4x^2y'' + \left(2x^4 - 5x\right) y' + \left(3x^2 + 2\right) y = 0.$$

22 Show that $\frac{d}{dx}(x\, J_1\, x) = x\, J_0\,(x)$.

23 Find the condition that $lx + my + nz + p = 0$. Should touch the central conicoid $ax^2 + by^2 + cz^2 = 1$.

24 Eliminate $a$ and $b$ from $2z = (ax+y)^2 + b$.

(5 × 1 = 5)

**Part C (Short Essay Questions)**

*Answer any* **four** *questions.*
*Each question has weight* 2.

25 Solve $x^2y - x^3 \frac{dy}{dx} = y^4 \cos x$.

26 Solve $\left(y - xy^2\right) dx - \left(x + x^2y\right) dy = 0$.

27 Solve the equation $y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x$.

28 Find the general solution of $\left(x^2 D^2 + 7xD + 9\right) y = 0$.

29 Find the general solution of $y'' + y = 0$ in terms of power series in $x$.

30 Find the general solution of the differential equation $x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x + y) z$.

(4 × 2 = 8)

## Part D (Essay Questions)

*Answer any* **two** *questions.*
*Each question has weight 4.*

31 Solve the equation $\left(3x + 2y^2\right) y \, dx + 2x\left(2x + 3y^2\right) dy = 0$.

32 Find a particular solution of $y' + 2y' + y = e^{-x} \log x$.

33 Verify that origin is a regular singular point of $4xy'' + 2y' + y = 0$ and calculate two independent Frobenius series solutions.

(2 × 4 = 8)