



23127310

QP CODE: 23127310

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE  
EXAMINATIONS, OCTOBER 2023**

**Third Semester**

**Core Course - MM3CRT01 - CALCULUS**

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc  
Mathematics Model II Computer Science

2017 Admission Onwards

AB9FB874

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Show that  $y = \log x$  is everywhere concave downwards.
2. Find the radius of the curvature  $y^2 = x^3$  at the point (4,8).
3. Define centre of curvature at any point p of a curve.
4. Find the envelope of the family of the semi-cubical parabola  $y^2 = (x+a)^2$ .
5. Evaluate  $f_{xy}$  if  $f(x, y) = \sqrt{x^2 + y^2}$
6. Find  $\frac{dw}{dt}$  if  $w = xy + z$ ,  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$
7. State the second derivative test for local maximum values of a function  $f(x, y)$  at  $(a, b)$ .
8. Evaluate the volume of the solid of cross sectional area  $A(x) = 5x^2$  from  $x = 0$  to  $x = 1$ .
9. Find the length of the curve  $y = x^{\frac{3}{2}}$  from  $x = 0$  to  $x = 4$ .
10. The line segment  $x = 1 - y$ ;  $0 \leq y \leq 1$  is revolved about the Y-axis to generate the cone. Find its lateral surface area (which excludes base area).
11. Find the average value of  $f(x, y) = \sin(x + y)$  over the rectangle  $0 \leq x \leq \pi$ ;  $0 \leq y \leq \pi$ .





12. Express the spherical coordinate  $(\rho, \phi, \theta)$  in terms of rectangular coordinates  $(x, y, z)$   
(10×2=20)

**Part B**

Answer any **six** questions.

Each question carries **5** marks.

13. Using Maclaurin's series expand  $\frac{e^x}{\cos x}$  around  $x=0$ .
14. Expand  $\log(x+k)$  in powers of  $x$  using Taylor's series.
15. If  $u = \ln \frac{x^2 + y^2}{x + y}$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
16. Find the absolute maximum and minimum values  $f(x, y) = 2 + 2x + 2y - x^2 - y^2$  on the triangular plate in the first quadrant bounded by the lines  $x = 0, y = 0, y = 9 - x$ .
17. The region between the curve  $y = \sqrt{x}$ ;  $0 \leq x \leq 4$  and the X-axis is revolved about the X-axis to generate a solid. Find its volume.
18. Find the volume of the solid generated by revolving each region in the first quadrant bounded above by the curve  $y = x^2$ , below by the X-axis and on the right by the line  $x = 1$ , about the line  $x = -1$ .
19. Sketch the region of integration and write an equivalent double integral of  $\int_0^2 \int_{x^2}^{2x} (4x + 3) dy dx$  with the order of integration reversed.
20. Find the average value  $f(x, y, z) = xyz$  over the cubical region  $D$  bounded by the coordinate planes  $x = 2, y = 2$  and  $z = 2$  in the first octant.
21. Find the Jacobian  $J(\rho, \phi, \theta)$  for the transformation  $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$ .

(6×5=30)

**Part C**

Answer any **two** questions.

Each question carries **15** marks.

22. Find all the asymptotes of the curve  $y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x - 1 = 0$
23. (a). Find the positive numbers  $x, y, z$  such that  $xyz = 64$  and  $x + y + z$  is minimum.  
(b). The plane  $x + y + z = 1$  cuts the cylinder  $x^2 + y^2 = 1$  in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.





24. (a). The region enclosed by the X-axis and the parabola  $y = 2x - x^2$  is revolved about the vertical line  $x = -1$  to generate a solid. Find the volume of the solid using shell method.

(b). Find the length of the curve  $y = \frac{x^3}{12} + \frac{1}{x}$  from  $x = 0$  to  $x = 4$ .

25. (a). Evaluate  $\int_0^1 \int_0^{1-x^2} \int_3^{(4-x^2-y)} x \, dz \, dy \, dx$

(b). Evaluate the cylindrical coordinate integral  $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} dz \, r \, dr \, d\theta$

(2×15=30)

