



23135620

QP CODE: 23135620

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER  
2023**

**Fifth Semester**

**CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS**

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc  
Computer Applications Model III Triple Main

2017 Admission Onwards

000B74C6

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the set of all integers  $\mathbb{Z}$  is denumerable.
2. Determine the set  $A = \{x \in \mathbb{R} : |2x + 3| < 7\}$ ?
3. If  $t > 0$  prove that there exist an  $n_t \in \mathbb{N}$  such that  $0 < \frac{1}{n_t} < t$
4. Justify the validity of the following statement with proper reasoning "A positive real number is rational then its decimal expansion is periodic".
5. Show that  $\lim(\frac{1}{n} - \frac{1}{n+1}) = 0$ .
6. Find  $\lim((2 + \frac{1}{n})^2)$ .
7. Use the recurrence relation of  $n^{\text{th}}$  term of a sequence that converges to  $\sqrt{a}$  to find the value of  $\sqrt{2}$  correct to 4 decimal places.
8. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
9. Prove that a monotone sequence of real numbers is properly divergent if and only if it is bounded.
10. Using Comparison test, discuss the convergence of  $\sum \frac{1}{n(n+1)}$ .





11. State Abel's test for the convergence of series.

12. Show that  $\lim_{x \rightarrow 0} (x + \operatorname{sgn}(x))$  do not exist.

(10×2=20)

### Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Prove that  $a < b \iff a^2 < b^2 \iff \sqrt{a} < \sqrt{b}, a, b \geq 0$ ,

14. State and prove characterisation of intervals theorem.

15. Using definition of limits, prove that  $\lim_{n \rightarrow \infty} \left( \frac{3n+2}{n+1} \right) = 3$ .

16. Let  $X = (x_n)$  and  $Y = (y_n)$  be sequences of real numbers that converges to  $x$  and  $y$  respectively and  $c \in \mathbf{R}$ . Prove that the sequences  $cX$  converges to  $cx$ .

17. Prove that every contractive sequence is Cauchy and hence is convergent.

18. State and prove the comparison test for the convergence of series. Using this test, show that  $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$  is convergent.

19. If  $\sum a_n$  is a convergent series of real numbers then is it necessary that  $\sum \frac{\sqrt{a_n}}{n}$  is convergent?

20. Evaluate the following one-sided limits

(a)  $\lim_{x \rightarrow 1^+} \frac{x}{x-1} (x \neq 1)$ .

(b)  $\lim_{x \rightarrow 0^+} \frac{(x+2)}{\sqrt{x}} (x > 0)$

21. Let  $A \subseteq \mathcal{R}$ ,  $f, g : A \rightarrow \mathcal{R}$ ,  $c \in \mathcal{R}$  be a cluster point of  $A$ . If  $f(x) \leq g(x)$  for all  $x \in A, x \neq c$ , Then prove the following

(a) If  $\lim_{x \rightarrow c} f = \infty$ , then  $\lim_{x \rightarrow c} g = \infty$ .

(b) If  $\lim_{x \rightarrow c} g = -\infty$ , then  $\lim_{x \rightarrow c} f = -\infty$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Prove that there exist a real number  $x$  such that  $x^2 = 2$  ?





23. (a) State and prove Monotone Convergence Theorem.

(b) Let  $Y = (y_n)$  be the sequence defined as  $y_1 = 1$  and  $y_{n+1} = \frac{2y_n + 3}{4}$ ,  $n \geq 1$ . Prove that  $\lim Y = \frac{3}{2}$ .

24.

1. State and prove Rearrangement Theorem.

2. If  $\sum a_n$  is convergent, then prove that any series obtained from it by grouping terms is also convergent to the same value.

25. (a) Let  $A \subseteq \mathcal{R}$ ,  $f : A \rightarrow \mathcal{R}$  and let  $c \in \mathcal{R}$  be a cluster point of  $A$ . If  $a \leq f(x) \leq b$  for all  $x \in A, x \neq c$ , and if  $\lim_{x \rightarrow c} f$  exists, Then prove that  $a \leq \lim_{x \rightarrow c} f \leq b$ .

(b) Check whether the following limits exist or not. Give explanations

$$(1) \lim_{x \rightarrow 0} \sin x \quad (2) \lim_{x \rightarrow 0} \left( \frac{\cos x - 1}{x} \right)$$

(2×15=30)

