



QP CODE: 24001052



24001052

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024**

**Sixth Semester**

**CORE COURSE - MM6CRT01 - REAL ANALYSIS**

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc  
Computer Applications Model III Triple Main

2017 Admission Onwards

25168F10

Time: 3 Hours

Max. Marks : 80

**Part A**

Answer any **ten** questions.

Each question carries **2** marks.

1. Let  $f$  be defined for all  $x \in \mathbb{R}, x \neq 2$  by  $f(x) = \frac{x^2+x-6}{x-2}$ . Define  $f$  at  $x = 2$  in such a way that  $f$  is continuous at that point.
2. Give an example of a function  $f : [0, 1] \rightarrow \mathbb{R}$  that is discontinuous at every point of  $[0, 1]$  but such that  $|f|$  is continuous on  $[0, 1]$ .
3. Define absolute maximum point and absolute minimum point for  $f : A \rightarrow \mathbb{R}$ .
4. Is every continuous function differentiable? Justify with proper reasoning or counter example.
5. Given that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^3 + 2x + 1$  is invertible and let  $g$  be its inverse. Find the value of  $g'(1)$ .
6. Define decreasing function with a proper example.
7. Define norm of the partition of an interval.
8. Test the function of  $f(x) = x^{2020} + 2021x$  on  $[2022, 2023]$  is Riemann integrable or not.
9. Under what circumstances differentiation and Riemann integration are inverse to each other.
10. Evaluate  $\lim_{x \rightarrow \infty} \left( \frac{\sin nx}{1+nx} \right)$  for  $x \in \mathbb{R}, x \geq 0$ .
11. Show that the sequence of functions  $f_n$  defined on  $\mathbb{R}$  as  $f_n(x) = \frac{\sin(nx+n)}{n}$  converges uniformly in  $\mathbb{R}$ .





12. Do the limit of a convergent sequence of differentiable functions on an interval  $[a, b]$  is differentiable, if not what condition will make the limit function differentiable?

(10×2=20)

### Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Define Thomae's function on  $(0, \infty)$  and show that it is continuous precisely at the irrational points in  $(0, \infty)$ .
14. State and prove Preservation of Intervals Theorem.
15. Let  $I \subseteq \mathbb{R}$  be an interval and let  $f : I \rightarrow \mathbb{R}$  be monotone on  $I$ . Then prove that the set of points  $D \subseteq I$  at which  $f$  is discontinuous is a countable set .
16. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \begin{cases} x^2, & x \text{ is rational} \\ x, & x \text{ is irrational} \end{cases}$ , Prove that  $f$  is differentiable at  $x = 0$ .
17. Derive the inequality  $x^\alpha \leq \alpha x + (1 - \alpha), \forall x \geq 0, 0 < \alpha < 1$
18. Evaluate the limit  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}, x \in (0, \infty)$
19. Evaluate  $\int_1^4 \frac{\sin \sqrt{t}}{\sqrt{t}} dt$ .
20. Evaluate  $\int_0^2 t^2 (1 + t^3)^{-\frac{1}{2}} dt$ .
21. Suppose that  $(f_n)$  is a sequence of continuous functions on an interval  $I$  that converges uniformly on  $I$  to a function  $f$ . If  $(x_n) \subseteq I$  converges to  $x_0 \in I$ , show that  $\lim(f_n(x_n)) = f(x_0)$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. (a) Show that a function  $f$  is uniformly continuous on the interval  $(a, b)$  if and only if it can be defined at the endpoints  $a$  and  $b$  such that the extended function is continuous on  $[a, b]$ .  
(b) State and prove the Continuous Inverse Theorem.
23. (a) State and Prove L'Hospital's Rule I  
(b) Using this, find the following





$$(i) \lim_{x \rightarrow 0^+} \frac{\tan x - x}{x^3}, x \in (0, \frac{\pi}{2})$$

$$(ii) \lim_{x \rightarrow 0^+} \frac{\log \cos x}{x}$$

24. (a) Let  $f \in \mathcal{R}[a, b]$  and if  $(\mathcal{P}_n)$  is any sequence of tagged partitions of  $[a, b]$  such that

$$||\mathcal{P}_n|| \rightarrow 0, \text{ prove that } \int_a^b f = \lim_n S(f; \mathcal{P}).$$

(b) Suppose that  $f$  is bounded on  $[a, b]$  and that there exists two sequences of tagged partitions  $(\mathcal{P}_n)$  and  $(\mathcal{Q}_n)$  of  $[a, b]$  such that  $||\mathcal{P}_n|| \rightarrow 0$  and  $||\mathcal{Q}_n|| \rightarrow 0$ , but such that  $\lim_n S(f; \mathcal{P}_n) \neq \lim_n S(f; \mathcal{Q}_n)$ . Show that  $f \notin \mathcal{R}[a, b]$ .

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function  $f : [a, b] \rightarrow \mathbb{R}$ .

(b) Check the Riemann integrability of Dirichlet function.

(2×15=30)

