



23105591

QP CODE: 23105591

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,  
MARCH 2023**

**Sixth Semester**

**CORE COURSE - MM6CRT04 - LINEAR ALGEBRA**

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

A2ADAE33

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that if the rows  $x_1, x_2, \dots, x_p$  are linearly independent, then none can be zero
2. Prove that every square matrix is equivalent to its transpose.
3. Define linearly dependent subset of a vector space  $V$ . Prove that  $\{(1, 1, 0), (2, 5, 3), (0, 1, 1)\}$  of  $\mathbb{R}^3$  is linearly dependent.
4. Check whether  $\{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$  is a basis of  $\mathbb{R}^3$ .
5. Define dimension of a vector space  $V$  and Find the dimension of  $\mathbb{R}_n[X]$
6. If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by  $f(a, b) = (b, 0)$ , prove that  $\text{Im } f = \text{Ker } f$ .
7. Define matrix of  $f$  relative to fixed ordered bases of vector spaces  $V$  and  $W$  where  $f : V \rightarrow W$  is linear.
8. Determine the transition matrix from the ordered basis  $\{(1, -1, 1), (1, -2, 2), (1, -2, 1)\}$  of  $\mathbb{R}^3$  to the natural ordered basis of  $\mathbb{R}^3$ .
9. Define a nilpotent linear mapping  $f$  on a vector space  $V$  of dimension  $n$  over a field  $F$ . What is meant by index of nilpotency of  $f$ .
10. Find the characteristic polynomial of  $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$





11. Define the eigen space and geometric multiplicity associated with the eigen value.
12. Define eigen value of a linear map and the eigen vector associated with it.

(10×2=20)

### Part B

Answer any **six** questions.

Each question carries **5** marks.

13. a) Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix  
b) If  $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$  Prove that  $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$
14. a) If A and B are orthogonal nxn matrices prove that AB is orthogonal.  
b) Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms  $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ ,  $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$  Where  $a^2 + b^2 = 1$ .
15. Prove that the set  $R^n$  of n tuples  $(x_1, x_2, \dots, x_n)$  of real numbers is a real vector space.
16. Determine which of the following subsets are subspace of a vector space  $R^4$   
a)  $\{ (x, y, z, t) : x = 1 \}$   
b)  $\{ (x, y, z, t) : x = y, z = t \}$
17. Define linear mapping from a vector space to a vector space. Check whether  $f : R^3 \rightarrow R^3$  given by  $f(x, y, z) = (z, -y, x)$  is linear.
18. a) Prove that the linear mapping  $f : R^2 \rightarrow R^3$  given by  $f(x, y) = (y, 0, x)$  is injective but not surjective.  
b) If  $f : V \rightarrow W$  is linear, then prove that the following statements are equivalent: (i)  $f$  is injective (ii)  $\text{Ker } f = \{0\}$ .
19. a) Define rank and nullity of a linear mapping. Find the rank and nullity of  $pr_1 : R^3 \rightarrow R$  defined by  $pr_1(x, y, z) = x$ .  
b) Let  $V$  and  $W$  be vector spaces each of dimension  $n$  over a field  $F$ . If  $f : V \rightarrow W$  is linear, then prove that  $f$  is injective if and only if  $f$  is bijective.
20. For the matrix  $A = \begin{bmatrix} -2 & 5 & 7 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$  find a matrix  $P$  such that  $P^{-1} A P$  is diagonal.





21.

For the  $n \times n$  tridiagonal matrix  $A_n = \begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$  Prove that  $\det A_n = n + 1$ .

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22.

a) Reduce the following matrix to row echelon form  $\begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$

b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

c) Prove that every non-zero matrix  $A$  can be transformed to a Hermite matrix by using elementary row operations.

23.

a) Show that the matrix  $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 3 & 5 & 8 \\ 1 & 4 & 5 & 9 \end{bmatrix}$  has neither a left inverse nor a right inverse.

b) Define an invertible matrix. Prove that if  $A$  and  $B$  are invertible matrix, then  $(AB)^{-1} = B^{-1}A^{-1}$ .

c) Prove that the real  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is invertible if and only if  $ad - bc \neq 0$ , in which case, find its inverse.

d) If  $A_1, A_2, \dots, A_p$  are invertible  $n \times n$  matrices. Prove that the product  $A_1 A_2 \dots A_p$  is invertible and that  $(A_1 A_2 \dots A_p)^{-1} = A_p^{-1} \dots A_2^{-1} A_1^{-1}$

24. a) Let  $V$  and  $W$  be vector spaces over a field  $F$ . If  $\{v_1, v_2, \dots, v_n\}$  is a basis of  $V$  and  $w_1, w_2, \dots, w_n$  are elements of  $W$  (not necessarily distinct) then prove that there is a unique linear mapping  $f: V \rightarrow W$  such that  $(i = 1, 2, \dots, n) \quad f(v_i) = w_i$ .
- b) Prove that a linear mapping is completely and uniquely determined by its action on a





basis.

c) Prove that two linear mappings  $f, g : V \rightarrow W$  are equal if and only if they agree on any basis of  $V$ .

25. a) Define similar matrices and state whether similar matrices have the same rank. Show that if matrices  $A, B$  are similar then so are  $A', B'$ .

b) Prove that for every  $\vartheta \in \mathbb{R}$ , the complex matrices  $\begin{bmatrix} \cos \vartheta & -\sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{bmatrix}$ ,

$\begin{bmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{bmatrix}$  are similar.

c) Prove that the relation of being similar is an equivalence relation on the set of  $n \times n$  matrices.

(2×15=30)

