



23104287

QP CODE: 23104287

Reg No :

Name :

**B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE
EXAMINATIONS, JANUARY 2023**

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc
Mathematics Model II Computer Science

2017 Admission Onwards

08C7E480

Time: 3 Hours

Max. Marks : 80

core

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Expand a^x by Maclaurin's series.
2. Find the points of inflection of the curve $y = x^3 - 3x^2 - 9x + 9$.
3. Find the radius of curvature at any point on the curve $s = a \log(\tan \psi + \sec \psi)$
4. Define centre of curvature at any point p of a curve.
5. State the three dimensional Laplace Equation .
6. By implicit differentiation find $\frac{dy}{dx}$ if $x^2 - \sin y + 2y = 0$
7. Explain the method of Lagrange multipliers to find the extreme values of a function $f(x, y, z)$ subject to a constraint $g(x, y, z) = 0$
8. The solid lies between planes perpendicular to the X-axis at $x = 0$ and $x = 4$. The cross-sections perpendicular to X-axis are squares with sides run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. Find the area of cross section $A(x)$.
9. Explain Shell formula for finding volume of solid obtained by revolving a bounded region about a vertical line .





10. Give the formula for finding the length of a smooth curve $y = f(x)$ from $x = a$ to $x = b$.
11. Calculate $\int \int_R f(x, y) dA$ where $f(x, y) = xy \cos y$ and $R: -1 \leq x \leq 1; 0 \leq y \leq \pi$
12. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \leq x \leq \pi; 0 \leq y \leq \pi/2$.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Using Taylor series expand $f(x) = \tan^{-1} x$ in powers of $(x - \frac{\pi}{4})$
14. Prove that the asymptotes of the curve $x^2 y^2 = c^2 (x^2 + y^2)$ are the sides of a square.
15. If $u = (y - z)(z - x)(x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
16. Find all local extreme values and saddle point, if any, of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
17. Find the volume of the solid generated by revolving the region bounded by the lines and curves $x = y^{\frac{3}{2}}; x = 0, y = 2$, about the Y-axis.
18. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the X-axis to generate a solid. Find the volume of the solid.
19. Sketch the region of integration and write an equivalent double integral of $\int_0^2 \int_{x^2}^{2x} (4x + 3) dy dx$ with the order of integration reversed.
20. Evaluate the spherical integral $\int_0^\pi \int_0^\pi \int_0^{2 \sin \phi} \rho^2 \sin \phi d\rho d\phi d\theta$.
21. Solve the system $u = 3x + 2y, v = x + 4y$ for x and y in terms of u and v .
Then find the value of the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$.

(6×5=30)

Part C





Answer any **two** questions.
Each question carries **15** marks.

22. a) Find all asymptotes of the cubic curve
 $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$
b) Find the envelope of the curve $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 1$ when $a^2 + b^2 = c^2$.
23. (a). Show that the function $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$ satisfies three dimensional Laplace equation.
(b). Find the absolute maximum and minimum values
 $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 2$, $y = 2x$.
24. (a). Using the shell method to find the volumes of the solids generated by revolving the regions bounded by the lines and curves
 $y = x + 2$; $y = x^2$ about (i) the X-axis (ii) the line $x = 2$.
(b). Find the area of the surface generated by revolving the curve
 $y = 2\sqrt{x}$; $1 \leq x \leq 2$ about the X-axis.
25. Find the volume of the region enclosed by the surface
 $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(2×15=30)

