E 7512 (Pages : 4) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014

Sixth Semester

Core Course—COMPLEX ANALYSIS

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions has weight* 1.

I. 1. What is an entire function ?

2. At which points the function $f(z) = \bar{z}$ differentiable ?

3. Define a Harmonic function.

4. What are the arguments of e^z ?

II. 5. Find the values of z such that $e^z = -2$.

6. Find $\log(-1)$.

7. Evaluate $\int_0^{\pi/6} e^{i2t}\, dt$.

8. State Cauchy-Goursat theorem.

III. 9. What do you mean by a simply connected domain.

10. If C is the positively oriented unit circle $|z| = 1$, then what is the value of $\int_C \frac{\exp(2z)}{z^4}\, dz$.

11. State Morera's theorem.

12. If C is any positively oriented simple closed curve surrounding the origin, then what is the value of $\int_C \frac{dz}{z}$?

Turn over

IV. 13. What is the Maclaurin's series expansion of $\frac{1}{1-z}$ if $|z|<1$?

14. Name the singularities of the function $\frac{1}{\sin(\pi/z)}$.

15. What is the order of the pole of $\frac{\sinh z}{z^4}$ at $z=0$?

16. Find the residue of $f(z)=\frac{1}{z^2(1+z)}$ at $z=0$.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17. Let $f(z) = \frac{z}{\bar{z}}$. Show that $\lim\limits_{z \to 0} f(z)$ doesnot exist.

18. Show that the function $f(z)=\operatorname{Im} z$ is no where differentiable.

19. Show that $f'(z)$ doesnot exist at any point for the function $f(z)=2x+ixy^2$.

20. Evaluate $\int\limits_C \frac{z+2}{z}\, dz$, where C is the semicircle $z=2e^{i\theta}$ $(0 \le \theta \le \pi)$.

21. Use Cauchy's integral formula to find $\int\limits_C \frac{z}{9-z^2}\, dz$, where C is the positively oriented circle $|z|=2$.

22. State Taylor's theorem.

23. What is the nature of the singularity at $z = 0$ for the function $\frac{1-\cos z}{z^2}$.

24. Show that the singular point of $\frac{\exp(2z)}{(z-1)^2}$ is a pole. Determine the order of the pole and the corresponding residue.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25. Suppose that $f(z) = u + iv$ and that $f'(z)$ exists at a point z_0. Show that the first order partial derivatives of u and v exists at (x_0, y_0) and that they satisfy Cauchy-Riemann equations.

26. Show that $u(x, y) = \sinh x \sin y$ is Harmonic in some domain and find a Harmonic conjugate $v(x, y)$.

27. State and prove fundamental theorem of algebra.

28. State and prove Cauchy's integral formula.

29. Derive the expansion

$$\frac{\sinh z}{z^2} = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+3)!}, \ (0 < |z| < \infty).$$

30. Evaluate $\int_C \frac{5z-2}{z(z-1)}\, dz$ using residue theorem.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31. (a) State and prove Liouville's theorem.

(b) If a function f is analytic throught a simply connected domain D then show that $\int_C f(z)\, dz = 0$ for every closed contour C lying in D.

32. Write two Laurent series in powers of Z that represent the function $f(z) = \frac{1}{z\left(1+z^2\right)}$ in certain domains and specify the domains.

33. Evaluate $\int_0^\infty \frac{x^2}{x^6+1}\, dx$.

(2 × 4 = 8)