



24001059

QP CODE: 24001059

Reg No :

Name :

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024**Sixth Semester****CORE COURSE - MM6CRT04 - LINEAR ALGEBRA**

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

6A418A62

Time: 3 Hours

Max. Marks : 80

Part A*Answer any **ten** questions.**Each question carries **2** marks.*

1. Define the Hermite matrix. Give an example of a Hermite matrix.
2. a) Define linearly dependent rows.
b) Prove that in the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ the columns are linearly dependent.
3. If V is a Vector space over a field F . Prove that a) $\forall \lambda \in F, \lambda 0 = 0$ b) $\forall x \in V, 0x = 0$
4. Prove that $\{(x, y, z, t) : x = y, z = t\}$ is a subspace of \mathbb{R}^4
5. Check whether $\{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$ is a basis of \mathbb{R}^3 .
6. If $f : V \rightarrow W$ is linear, X is a subset of V and Y is a subset of W , define direct image of X under f and inverse image of Y under f .
7. Determine the transition matrix from the ordered basis $\{(1, 0, 0, 1), (0, 0, 0, 1), (1, 1, 0, 0), (0, 1, 1, 0)\}$ of \mathbb{R}^4 to the natural ordered basis of \mathbb{R}^4 .
8. a) Define similar matrices.
b) "Similar matrices have the same rank"-True or False?
9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F . What is meant by index of nilpotency of f .



10. Find the eigen values of $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

11. Define eigen value of a linear map and the eigen vector associated with it.

12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. a) Prove that addition of matrices is associative.

b) Write 3x3 matrix whose entries are given by $x_{ij} = (-1)^{i-j}$

14. a) If A and B are orthogonal nxn matrices prove that AB is orthogonal.

b) Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$,
 $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ Where $a^2 + b^2 = 1$.

15. a) Define span S of a vector space V and Prove that $S = \{(1,0), (0,1)\}$ is a spanning set of R^2

b) Prove that $\{(1,1,0), (2,5,3), (0,1,1)\}$ of R^3 is linearly dependent.

16. If S is a subset of V, then prove that S is a basis if and only if S is a maximal independent subset.

17. Define $Im f$ and $Ker f$ where f is a linear mapping from a vector space to a vector space. Write image and kernel for the i-th projection of R^n onto R .

18. Define injective linear mapping. Prove that if the linear mapping $f : V \rightarrow W$ is injective and $\{v_1, v_2, \dots, v_n\}$ is a linearly independent subset of V then $\{f(v_1), f(v_2), \dots, f(v_n)\}$ is a linearly independent subset of W .

19. a) Let V be a vector space of dimension $n \geq 1$ over a field F . Then prove that V is isomorphic to the vector space F^n .

b) If V and W are vector spaces of the same dimension n over F , then prove that V and W are isomorphic.

20. Determine the eigen values and their algebraic multiplicities of the linear mapping $f: R^3 \rightarrow R^3$ given by $f(x,y,z) = (x+ 2y + 2z, 2y + z, -x + 2y+ 2z)$



21.

For the $n \times n$ tridiagonal matrix $A_n =$

$$\begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$$

Prove that $\det A_n = n + 1$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. a) Prove that if A is an $m \times n$ matrix then the homogeneous system of equation $Ax = 0$ has a nontrivial solution if and only if $\text{rank } A < n$.

b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ is of rank 3 and find matrices P, Q such that

$$PAQ = [I_3, 0].$$

- c) Show that the system of equations $x + y + z + t = 4$, $x + \beta y + z + t = 4$, $x + y + \beta z + (3 - \beta)t = 6$, $2x + 2y + 2z + \beta t = 6$ has a unique solution if $\beta \neq 1, 2$.

23. a) Define a left inverse and right inverse of a matrix.

b) Prove that the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 3 \end{bmatrix}$ has a common unique left inverse and unique right inverse.

c) Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

- d) If A_1, A_2, \dots, A_p are invertible $n \times n$ matrices, prove that the product $A_1 A_2 \dots A_p$ is invertible and that $(A_1 A_2 \dots A_p)^{-1} = A_p^{-1} \dots A_2^{-1} A_1^{-1}$

24. Let V and W be vector spaces each of dimension n over a field F . If $f : V \rightarrow W$ is linear then prove that the following statements are equivalent:

- (i) f is injective (ii) f is surjective (iii) f is bijective (iv) f carries bases to bases, in the sense that if $\{v_1, \dots, v_n\}$ is a basis of V then $\{f(v_1), \dots, f(v_n)\}$ is a basis of W .



25. A linear mapping $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is such that $f(1, 0, 0) = (0, 0, 1)$, $f(1, 1, 0) = (0, 1, 1)$, $f(1, 1, 1) = (1, 1, 1)$. Determine $f(x, y, z)$ for all $(x, y, z) \in \mathbb{R}^3$ and compute the matrix of f relative to the ordered basis $B = \{(1, 2, 0), (2, 1, 0), (0, 2, 1)\}$. If $g : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is the linear mapping given by $g(x, y, z) = (2x, y + z, -x)$, compute the matrix $f \circ g \circ f$ relative to the ordered basis B .

(2×15=30)

