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QP CODE: 22101917

Reg No :

Name :

**B.Sc DEGREE (CBCS) SPECIAL SUPPLEMENTARY EXAMINATIONS,
MAY 2022**

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2019 Admission Only

3B967B88

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define identity element for $*$ in a binary structure $\langle S, * \rangle$.
2. Define an abelian group.
3. Define generator for a group.
4. Compute $\sigma^{-1}\tau\sigma$ involving the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ in S_6 .
5. Define a **cycle**. Write the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ in S_8 as a product of disjoint cycles.
6. Define a **transposition**. Show that any cycle is a product of transpositions.
7. Let H be a subgroup of a group G . Define the index $(G : H)$ of H in G . Give a formula to compute $(G : H)$ when G is finite.
8. Check whether $f : (GL_n(\mathbb{R}), \cdot) \rightarrow (\mathbb{R}^*, \cdot)$ defined by $f(A) = \det(A)$ is a group homomorphism or not.
9. Define simple group with an example.
10. Define
 - a) ring homomorphism
 - b) ring isomorphism
11. Find all solutions of the equation $x^2 + 2x + 4 = 0$ in Z_6 .
12. Prove that Z_p is a field if p is a prime.

(10×2=20)





Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Let S be a set and let f, g and h be functions mapping S into S . Then prove that $f \circ (g \circ h) = (f \circ g) \circ h$.
14. Let G be a group with binary operation $*$. Then prove that the linear equations $a * x = b$ and $y * a = b$ have unique solutions x and y in G , where a and b are any elements of G .
15. Find the quotient q and remainder r when -38 is divided by 7 according to the division algorithm.
16. Let G and G' be groups and let $\phi : G \rightarrow G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$. Then show that $\phi[G]$ is a subgroup of G' and ϕ provides an isomorphism of G with $\phi[G]$.
17. Let G_1, G_2, \dots, G_n be groups. For (a_1, a_2, \dots, a_n) and (b_1, b_2, \dots, b_n) in $\prod_{i=1}^n G_i$, define $(a_1, a_2, \dots, a_n)(b_1, b_2, \dots, b_n)$ to be the element $(a_1 b_1, a_2 b_2, \dots, a_n b_n)$. Then prove that $\prod_{i=1}^n G_i$ is a group under this binary operation.
18. Show that composition of group homomorphisms is again a group homomorphism.
19. Show that intersection of normal subgroups of a group G is a normal subgroup of G .
20. Show that if a and b are nilpotent elements of a commutative ring, then $a + b$ is also nilpotent.
21. Let N be an ideal of a ring R . Prove that $\gamma : R \rightarrow R/N$ given by $\gamma(x) = x + N$ is a ring homomorphism with kernel N .

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. a) Prove that a subset H of a group G is a subgroup of G if and only if
 - i) H is closed under the binary operation of G ,
 - ii) the identity element e of G is in H ,
 - iii) for all $a \in H$ it is true that $a^{-1} \in H$ also.
 b) Let G be the multiplicative group of all invertible $n \times n$ matrices with entries in \mathbb{C} and let T be the subset of G consisting of those matrices with determinant 1. Then prove that T is a subgroup of G .
23. 1. Let H be a subgroup of a group G . Let the relation \sim_L be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Then show that \sim_L is an equivalence relation on G . What is the cell in the corresponding partition of G containing $a \in G$?





2. Let H be a subgroup of a group G . Then define the left and right cosets of H containing $a \in G$.
3. Let H be the subgroup $\langle \mu_1 \rangle = \{\rho_0, \mu_1\}$ of S_3 . Find the partitions of S_3 into right cosets of H .
24. State and prove fundamental homomorphism theorem.
25. a) Prove that a ring homomorphism $\phi : R \rightarrow R'$ is a one to one map if and only if $\ker \phi = \{0\}$
b) Show that an intersection of ideals of a ring R is again an ideal of R
c) Find all ideals N of Z_{12}

(2×15=30)

