



QP CODE: 23135626

Reg No :

Name :

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER
2023**

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

C91B8DE2

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State whether the set \mathbb{Z}^+ under multiplication is a group. Justify.
2. Define left identity element in a group and left inverse of an element in a group.
3. Define a cyclic group.
4. Define a **permutation of a set**. Compute $\sigma\tau$ where σ and τ are permutations given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}.$$
5. Define the **orbits** of a permutation of a set.
6. Let H be a subgroup of a group G . Define the index $(G : H)$ of H in G . Give a formula to compute $(G : H)$ when G is finite.
7. Define the **Cartesian product of sets** S_1, S_2, \dots, S_n . Write the number of elements in the Cartesian product of the sets $\{0, 1\}$ and $\{0, 1, 2\}$.
8. Check whether $f : (GL_n(\mathbb{R}), \cdot) \rightarrow (\mathbb{R}^*, \cdot)$ defined by $f(A) = \det(A)$ is a group homomorphism or not.
9. If $\phi : G \rightarrow G'$ is a group homomorphism then show that $\phi(e) = e'$ where e and e' are identity elements of G and G' respectively.
10. Compute the product in the given ring a) $(2,3)(3,5)$ in $Z_5 \times Z_9$ b) $(-3,5)(2,-4)$ in $Z_4 \times Z_{11}$
11. Find all units in Z_{14} .





12. Give an example to show that a factor ring of an integral domain may be a field.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Determine whether $*$ defined on \mathbb{Z}^+ by $a * b = a^b$ is a) commutative b) associative.
14. Check whether $\langle \mathbb{Q}, + \rangle$ and $\langle \mathbb{Z}, + \rangle$ under the usual addition are isomorphic.
15. State and prove Division Algorithm for \mathbb{Z} .
16. Let G and G' be groups and let $\phi : G \rightarrow G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$. Then show that $\phi[G]$ is a subgroup of G' and ϕ provides an isomorphism of G with $\phi[G]$.
17. If $n \geq 2$, then prove that the collection of all even permutations of $\{1, 2, 3, \dots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n .
18. Let $\phi : G \rightarrow G'$ is a group homomorphism, if N is a normal subgroup of G' show that $\phi^{-1}(N)$ is a normal subgroup of G .
19. Define maximal normal subgroup of a group. Prove that M is a maximal normal subgroup of a group G if and only if the factor group G/M is simple.
20. a) Mark each of the following true or false.
i) Every field is an integral domain
ii) The characteristic of $n\mathbb{Z}$ is n
b) Prove that \mathbb{Z}_p is a field if p is a prime.
21. Show that if R, R' and R'' are rings and if $\phi : R \rightarrow R'$ and $\tau : R' \rightarrow R''$ are homomorphism, then the composite function $\tau\phi : R \rightarrow R''$ is a homomorphism.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Find all subgroups of \mathbb{Z}_{36} and draw the subgroup diagram.

23.

1. Let H be a subgroup of a group G . Let the relation \sim_R be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that \sim_R is an equivalence relation on G . What is the cell in the corresponding partition of G containing $a \in G$?





2. Let H be a subgroup of a group G . Then define the left and right cosets of H containing $a \in G$.
3. Let H be the subgroup $\langle \mu_1 \rangle = \{\rho_0, \mu_1\}$ of S_3 . Find the partitions of S_3 into left cosets of H .
24. State and prove fundamental homomorphism theorem.
25. a) Prove that the divisors of 0 in Z_n are those nonzero elements that are not relatively prime to n .
b) Find the divisors of Z_{16}
c) Prove that Z_p , where p is prime has no divisors of 0.

(2×15=30)

