

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015**Fourth Semester****Complementary Course—OPERATIONS RESEARCH—NON-LINEAR PROGRAMMING****(For B.Sc. Mathematics (Model II))****[2013 Admissions]****Time : Three Hours****Maximum : 80 Marks****Part A***Answer all questions. Each question carries 1 mark.*

1. What is a pure integer programming problem ?
2. Define a Zero-one integer programming problem.
3. How does an integer programming problem differ from a L.P.P ?
4. When a subproblem is said to be fathomed in branch and bound method.
5. What is the effect of the 'integer' restriction of all the variables on the feasible space of integer programming problem ?
6. Define a Convex Programming.
7. What is a non-linear programming problem ?
8. Define separable function.
9. Write the Lagrangian function for the problem

$$\text{Minimize } (x_1 + 1)^2 + (x_2 - 2)^2$$

$$\text{subject to } x_1 - 2 \leq 0$$

$$x_2 - 1 \leq 0$$

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

10. Define saddle point.

(10 × 1 = 10)**Part B***Answer any eight questions. Each question carries 2 marks.*

11. What are the disadvantages of cutting plane method ?
12. Explain whether an integer programming can be solved by rounding off the corresponding simplex solution.

Turn over

13. Prove that if an optimal solution of the related L.P. problem of an integer or mixed integer programming problem is an integer or mixed integer vector, then it is also an optimal solution of the problem.
14. Explain how Gomory's cutting plane algorithm works.
15. Discuss the advantages of branch and bound method.
16. If $F(X, Y)$ has a saddle point (X_0, Y_0) for every $Y \geq 0$, then show that X_0 is a minimal point of $f(X)$ subject to the constraints $G(X) \leq 0$, where $F(X, Y) = f(X) + Y'G(X)$, and $G(X) = [g_1(X) \ g_2(X) \ \dots \ g_n(X)]$
17. Write down the Kuhn-Tucker conditions.
18. Maximize x^4 subject to $\frac{-1}{2} \leq x \leq 1$.
19. Write the Kuhn-Tucker conditions for
- $$\text{Minimize } f = (x_1 - 2)^2 + x_2^2$$
- $$\text{subject to } x_1^2 + x_2 \leq 1$$
- $$x_2 \geq 0.$$

20. Define a separable programming problem.
21. Use the method of Lagrangian multipliers to solve, optimize
- $$z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100.$$
- subject to the constraints $g(x) = x_1 + x_2 + x_3 = 20$ and $x_1, x_2, x_3 \geq 0$.
22. Discuss the primal and dual problems associated with non-linear programming.

(8 × 2 = 16)

Part C

Answer any six questions. Each question carries 4 marks.

23. Prove that if an optimal solution of
$$\left. \begin{array}{l} \text{Minimize } f(X) = CX \\ \text{subject to } X \in S_F \end{array} \right\}$$

exists and T_F is non-empty, then optimal solutions of,
$$\left. \begin{array}{l} \text{Minimize } f(X) = CX, \\ \text{subject to } X \in T_F \end{array} \right\} \text{ and}$$

$$\left. \begin{array}{l} \text{Minimize } f(X) = CX, \\ \text{subject to } X \in [T_F] \end{array} \right\} \text{ exist.}$$

24. Solve by cutting plane method

$$\text{Minimize } 4x_1 + 5x_2$$

$$\text{subject to } 3x_1 + x_2 \geq 2$$

$$x_1 + 4x_2 \geq 5$$

$$3x_1 + 2x_2 \geq 7$$

$$x_1, x_2$$

non-negative integers.

25. Give a brief note on branch and bound method.

26. Solve by branch and bound method

$$\text{Minimize } 9x_1 + 10x_2$$

$$\text{subject to } 0 \leq x_1 \leq 10$$

$$0 \leq x_2 \leq 8$$

$$3x_1 + 5x_2 \geq 45,$$

$$x_2 \text{ integer.}$$

27. Formulate the following knapsack problem as an ILP.

There are n objects, $j = 1, 2, \dots, n$ whose weight are w_j and values v_j . They have to be chosen to be packed in a knapsack so that the total value of the objects chosen is maximum subject to their total weight not exceeding W .

28. Solve graphically,

$$\text{Minimize } (x_1 - 2)^2 + (x_2 - 3)^2$$

$$\text{subject to } x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 1$$

$$2x_1 + x_2 \geq 6$$

$$\frac{1}{2}x_1 - x_2 \geq -4$$

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

29. Show that K-T conditions fails to give

$$\text{Maximize } x_1.$$

$$\text{subject to } (1 - x_1)^3 - x_2 \geq 0$$

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

30. Use K-T conditions to find the

$$\text{Minima and maxima of } (x_1 - 4)^2 + (x_2 - 3)^2$$

$$\text{subject to } 36(x_1 - 2)^2 + (x_2 - 3)^2 \leq 9.$$

Turn over

31. Mark on the graph the set of feasible solutions of

$$(x_1 - 1)(x_2 - 1) \leq 1$$

$$x_1 + x_2 \geq 6$$

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

(6 × 4 = 24)

Part D

Answer any **two** questions.
Each question carries 15 marks.

32. Maximize $2x_1 + 5x_2$

subject to $0 \leq x_1 \leq 8$

$$0 \leq x_2 \leq 8$$

and either $4 - x_1 \geq 0$ or $4 - x_2 \geq 0$.

33. Solve the knapsack problem with the following data :

| Object | | Weight | Value |
|--------|-----|--------|-------|
| j | ... | w_j | v_j |
| 1 | ... | 3 | 12 |
| 2 | ... | 4 | 12 |
| 3 | ... | 3 | 9 |
| 4 | ... | 6 | 30 |
| 5 | ... | 10 | 20 |
| 6 | ... | 12 | 12 |

knapsack capacity $W = 14$.

34. Solve by the method of quadratic programming,

$$\text{Minimize } -6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$$

subject to $x_1 + x_2 \leq 2$.

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

35. Solve using separable programming technique,

$$\text{Minimize } f = 2x_1 - 3x_2$$

subject to $4x_1^2 + 9x_2^2 \leq 36$

$$x_1 \geq 0$$

$$x_2 \geq 0.$$

(2 × 15 = 30)