E 2447

(Pages : 4)

Reg. No.................................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2016

Second Semester

Complementary Course—Mathematics

INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A

Answer all questions.
Each question carries 1 mark.

1. Evaluate $\int \cos \frac{x}{2} \, dx$.

2. State mean value theorem for definite integrals.

3. Evaluate $\int_0^\pi (1 + \cos x) \, dx$.

4. Write the formula for calculating the volume of a solid of revolution about the x-axis.

5. Write the surface area formula for revolution about the x-axis.

6. Find the area of the region bounded by $y = x$ and $y = x^2$ in the first quadrant.

7. How to change Cartesian integrals into polar integrals ?

8. State Cayley-Hamilton theorem.

9. What is the rank of a non-singular matrix of order n.

10. What are the elementary row transformations ?

$(10 \times 1 = 10)$

Part B

Answer any eight questions.
Each question carries 2 marks.

11. Evaluate $\int \frac{1}{t^2} \cos \left(\frac{1}{t} - 1 \right) dt$.

Turn over

12. Evaluate $\int\limits_{-4}^{4} |x|\, dx$.

13. Suppose f is continuous and that $\int_0^3 f(z)\, dz = 3$ and $\int_0^4 f(z)\, dz = 7$. Find $\int\limits_4^3 f(t)\, dt$.

14. Find the area between the curves $y = x^2$ and $y = -x^2 + 4x$.

15. Find the volume of the solid generated by revolving $y = x - x^2$, $y = 0$ about x-axis.

16. Find the length of the curve $y = \dfrac{4\sqrt{2}}{3} x^{3/2} - 1, 0 \le x \le 1$.

17. Evaluate $\int\limits_0^1 \int\limits_y^{\sqrt{y}} dx\, dy$.

18. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

19. Evaluate $\int_0^2 \int_0^1 \int_0^{1-z} dy\, dz\, dx$.

20. By reducing to the normal form, find the rank of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}$.

21. If A is an $n \times n$ non-singular matrix. Show that $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

22. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find A^{-1}.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Express the solution of the initial value problem $\frac{dy}{dx} = \sqrt{1+x^2}$, $y(1) = -2$ in terms of integrals.

24. Find $\frac{dy}{dx}$ where $y = \int\limits_0^{\sqrt{x}} \sin\left(t^2\right) dt$.

25. Find the area of the region in the 1st quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$, by integrating with respect to y.

26. A pyramid 3 m. high has a square base that is 3 m. on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

27. Calculate $\iint\limits_R \frac{\sin x}{x} \, dA$ where R is the triangle in the xy-plane bounded by the x-axis, the line

    $y = x$ and the line $x = 1$.

28. Sketch the region bounded by lines $x = 0$, $y = 2x$ and $y = 4$. Express the region's area as an iterated double integral and evaluate the integral.

29. Evaluate using polar integrals $\int\limits_{-1}^{1} \int\limits_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx$.

30. Prove that the rank of the transpose of a matrix is same as that of the original matrix.

31. Obtain the row equivalent canonical matrix of $\begin{bmatrix} 1 & 2 & -3 & -4 & 6 \\ 1 & 3 & 1 & -2 & 4 \\ 2 & 5 & -2 & -5 & 10 \end{bmatrix}$.

$(6 \times 4 = 24)$

Turn over

Part D

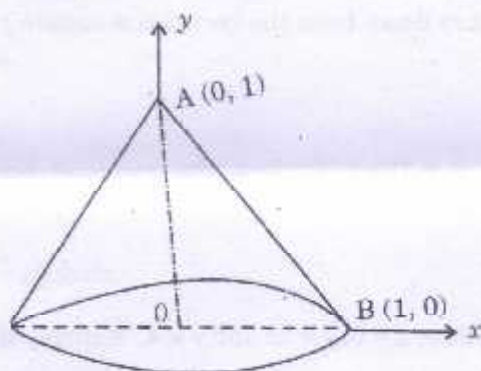*Answer any two questions.*
Each question carries 15 marks.

32. (a) Find the area of the region common to the interiors of the cardiods $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

    (b) Find the volume of the region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

33. (a) Evaluate $\int \dfrac{1}{\sqrt{x}\left(1 + \sqrt{x}\right)^2}\, dx$.

    (b) The line segment $x = 1 - y, 0 \leq y \leq 1$ is revolved about the y-axis to generated the cone as in the figure below :

    Find its lateral surface area.

34. (a) Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

    (b) Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.

35. (a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.

    (b) Solve the system of equations :

    $5x + 3y + 3z = 48$

    $2x + 6y - 3z = 18$

    $8x - 3y + 2z = 21$.

$(2 \times 15 = 30)$