

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016**Third Semester**

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for B.Sc. Physics, Chemistry, Petrochemicals Geology Computer Maintenance and Electronics and Food Science and Quality Control)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

Part A*Answer all questions from this part.**Each question carries 1 mark.*

1. Show that curvature of a straight line is zero.
2. $r(t) = 3 \cos t \, i + 3 \sin t \, j + t^2 \, k$ gives the position of a moving body at time t . Find the acceleration of the body at time t .
3. Define gradient field of a differentiable function $f(x, y, z)$.
4. Define a conservation field.
5. State normal form of Green's theorem.
6. How will you define flux of a three dimensional vector field across on oriented surface S ?
7. Find an integrating factor of the differential equation $(y - 2x^3) dx - x(1 - xy) dy = 0$.
8. Define an exact differential equation.
9. Find the asymptotes of the hyperbola $\frac{y^2}{4} - \frac{x^2}{5} = 1$.
10. Find the eccentricity of the ellipse $9x^2 + 10y^2 = 90$.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** questions.

Each question carries 2 marks.

11. Find the unit tangent vector to the curve $r(t) = t \cos t \mathbf{i} + t \sin t \mathbf{j} + \left(2\sqrt{2}/3\right) t^{3/2} \mathbf{k}$, $0 \leq t \leq \pi$.
12. Find the binomial vector \mathbf{B} for the curve $r(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j} + 3\bar{\mathbf{k}}$.
13. Find ∇f at $(1, 1, 1)$ where $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z + \tan^{-1} xz$.
14. Evaluate $\int_C \sqrt{x^2 + y^2} ds$ along the curve $r(t) = (4 \cos t) \mathbf{i} + (4 \sin t) \mathbf{j} + 3t \mathbf{k}$, $-2\pi \leq t \leq 2\pi$.
15. Find the work done by $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ over the curve $r(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq 1$.
16. Find the curl of $\mathbf{F} = (x^2 - y)\mathbf{i} + 4z\mathbf{j} + x^2\mathbf{k}$.
17. Solve the equation $p^2 + 2py \cot x = y^2$, where $p = \frac{dy}{dx}$.
18. Solve $\frac{dy}{dx} + y \tan x = \sec x$.
19. Solve $y = p \sin p + \cos p$.
20. Find the focus and directrix of the parabola $y = -8x^2$.
21. Sketch the ellipse $2x^2 + y^2 = 2$.
22. Find the polar equation of the circle $(x - 6)^2 + y^2 = 36$.

(8 × 2 = 16)

Part C*Answer any six questions.**Each question carries 4 marks.*

23. Find the curvature for the helix $r(t) = (a \cos t)i + (a \sin t)j + bt\bar{k}$, $a, b \geq 0$, $a^2 + b^2 \neq 0$.
24. Estimate how much the value of $f(x, y, z) = xe^y + yz$ will change if the point $p(x, y, z)$ moves 0.1 unit from $p_0(2, 0, 0)$ straight toward $p_1(4, 1, -2)$.
25. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the xy plane.
26. Find the flux of $F = yzj + z^2k$ outward through the surface s cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$ by the planes $x = 0$ and $x = 1$.
27. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
28. Solve $y^2(y - xp) = x^4 p^2$.
29. Solve $(\cos x - x \cos y) \frac{dy}{dx} = \sin y + y \sin x$.
30. Derive the standard equation of the parabola.
31. Sketch the hyperbola $8y^2 - 2x^2 = 16$ include asymptotes and foci in your sketch.

 $(6 \times 4 = 24)$ **Part D***Answer any two questions.**Each question carries 15 marks.*

32. (a) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $p_0(1, -1, 2)$ in the direction of $A = 3i + 6j - 2k$.
- (b) Write a in the form $a = a_T T + a_N N$ at the given value of t without finding T and N :
 $r(t) = t \cos t i + t \sin t j + t^2 k, t = 0$.

Turn over

33. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \geq 0$, by the cylinder $x^2 + y^2 = 1$.
34. Use the surface integral in Stoke's theorem to calculate the circulation of the field $\mathbf{F} = 2y\mathbf{i} + 3x\mathbf{j} - z^2\mathbf{k}$ around the curve C : the circle $x^2 + y^2 = 9$ in the xy plane, counter clockwise when viewed from above.
35. (a) Sketch the conic $r = \frac{6}{2 + \cos \theta}$.
- (b) Find a polar equation for an ellipse with semimajor axis 39.44 AU and eccentricity 0.25.

(2 × 15 = 30)