

C.B.C.S.S. - B.Sc. DEGREE EXAMINATION, APRIL 2011

Second Semester

INTEGRAL CALCULUS AND MATRICES

(Complementary Course to Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

Time : Three Hours

Maximum Weight : 25

Part A

Answer all questions.

A bunch of 4 questions has weight 1.

I. 1 Express the limit $\lim_{|P| \rightarrow 0} \sum_{k=1}^n C_k^2 \Delta x_k$, where P is a partition of $[0, 2]$ as definite integral.

2 Suppose that $\int_{-3}^0 g(t) dt = \sqrt{2}$. Find $\int_{-3}^0 \frac{g(r)}{\sqrt{2}} dr$.

3 Find $\int_{-\pi/4}^0 \sec x \tan x dx$.

4 Find $\frac{dy}{dx}$ if $y = \int_0^x \sqrt{1+t^2} dt$.

II. 5 State the Max-Min inequality for definite integrals.

6 Give an example of an odd function defined on the real line.

7 Define a solid of revolution.

8 Find the length of the curve :

$$x = 1-t, y = 2+3t, \frac{-2}{3} \leq t \leq 1.$$

III. 9 Give an example of a smooth curve.

10 Find $\int_{-1}^0 \int_{-1}^1 (x+y+1) dx dy$.

Turn over

- 11 Sketch the region bounded by the coordinate axes and the line $x + y = 2$.
- 12 Express the area of a closed and bounded region R in the polar coordinate plane as a double integral.

IV. 13 Find the rank of :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 \end{bmatrix}$$

- 14 What are the characteristic values of the zero matrix of order 3×3 ?
- 15 What is the degree of the characteristic polynomial of A , if A is an $n \times n$ matrix ?
- 16 Write an elementary matrix obtained from :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

($4 \times 1 = 4$)

Part B

Answer any **five** questions.

Each question has weight 1.

- 17 Find the average value of $f(x) = 3x^2 - 3$ on $[0, 1]$.
- 18 Find a function $y = f(x)$ on the domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with derivative $\frac{dy}{dx} = \tan x$ that satisfies the condition $y(3) = 5$.
- 19 Evaluate $\int_{\pi/4}^{\pi/2} \cot \theta \operatorname{cosec}^2 \theta d\theta$.
- 20 The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the X-axis is revolved about the X-axis to generate a solid. Find its volume.
- 21 Calculate $\iint_R f(x, y) dA$ for $f(x, y) = 1 - 6x^2y$ and $R: 0 \leq x \leq 2, -1 \leq y \leq 1$.

- 22 Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \leq x \leq \pi, 0 \leq y \leq \pi$.
- 23 What are the elementary transformations on a matrix?
- 24 Find the characteristic polynomial of:

$$B = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & 2 \end{pmatrix}.$$

(5 × 1 = 5)

Part C

*Answer any four questions.
Each question has weight 2.*

- 25 Find the area of the region between the X-axis and the graph of:

$$f(x) = x^3 - x^2 - 2x; -1 \leq x \leq 2.$$

- 26 Use substitution to evaluate the integral, $\int_0^{2\pi} \frac{\cos x}{\sqrt{4 + 3 \sin x}} dx$.

- 27 The circle $x^2 + y^2 = a^2$ is rotated about the x-axis to generate a sphere. Find its volume.

- 28 Find:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2) dx dy.$$

- 29 Converting to a polar integral, evaluate $\iint_R \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} dR$, over the region $1 \leq x^2 + y^2 \leq e^2$.

- 30 Find the characteristic values of the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ and the associated eigen-

vectors.

(4 × 2 = 8)

Turn over

Part D

*Answer any two questions.
Each question has weight 4.*

31 Find the area of the surface swept out by revolving the circle $x^2 + y^2 = 1$ about the x -axis.

32 Evaluate the integral :

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz \, dy \, dx.$$

33 Given :

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

Use Cayley-Hamilton theorem to compute A^3 , A^4 , A^{-1} and A^{-2} .

(2 × 4 = 8)