

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012**Third Semester****Complementary Course—VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY**

[Common for (1) Physics, (2) Chemistry, (3) Petrochemicals, (4) Geology,
(5) Computer Maintenance and Electronics, (6) Food Science and Quality Control]

[For 2011 Admission onwards]

Time : Three Hours

Maximum Weight : 25

Part A

Answer all questions.

Each bunch of four questions carries weight 1.

- I. 1 Define continuity of a vector function $r(t)$ at a point.
- 2 Find the length of one turn of the helix $r(t) = \cos t \, i + \sin t \, j + t \, k$.
- 3 What is the curvature of a straight line?
- 4 Define gradient of $f(x, y)$ at a point.
- II. 5 Write the formula for calculating flux across a smooth closed plane curve.
- 6 Whether the field $F = yz\bar{i} + xz\bar{j} + xy\bar{k}$, conservative.
- 7 Find the divergence of :

$$F(x, y) = (x^3 - y^2)\bar{i} + (xy - y)\bar{j}$$
- 8 State Stoke's theorem.
- III. 9 Is the equation $(e^x + 1) \cos x \, dx + e^x \sin x \, dy = 0$ exact.
- 10 Which type of equations are called Clairaut's equations.
- 11 Write the general form of Bernoulli's equation.
- 12 Write the integrating factor of $(y - x^2) \, dx + (x^2 \sin y - x) \, dy = 0$.
- IV. 13 Find the directrix of the parabola $x = 2y^2$.
- 14 Write the equation of the hyperbola in standard form where Foci $(0, \pm\sqrt{2})$ and asymptotes $(0, \pm\sqrt{5})$.
- 15 Find the eccentricity of the hyperbola $y^2 - 3x^2 = 3$.
- 16 Which curve is represented by the equation $x^2 + 2xy + y^2 + 2x - y + 2 = 0$.

(4 × 1 = 4)

Turn over

Part B

Answer any **five** questions.
Each question has weight 1.

- 17 Find the point on the curve $r(t) = 12 \sin t \mathbf{i} - 12 \cos t \mathbf{j} + 5t \mathbf{k}$ at a distance 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.
- 18 What is a differentiable curves unit tangent vector? Give *one* example.
- 19 Evaluate $\int_c (x+y) dS$, where c is the straight line segment $x=t, y=1-t, z=0$ from $(0, 1, 0)$ to $(1, 0, 0)$.
- 20 Find the work done by the force $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ over the curve $r(t) = t\mathbf{i} + t^2\mathbf{j} - t\mathbf{k}$, $0 \leq t \leq 1$.
- 21 Solve the equation $p^2 - 5p + 6 = 0$.
- 22 Solve the equation :

$$y - x \frac{dy}{dx} = x + y \frac{dy}{dx}.$$

- 23 The ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ is shifted 3 units to the left and 2 units down to generate the ellipse $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{25} = 1$. Find the foci, vertices and center of the new ellipse.
- 24 The x and y axes are rotated through an angle $\pi/4$ radians about the origin. Find an equation for the hyperbola $2xy = 9$ in the new co-ordinates.

(5 × 1 = 5)

Part C

Answer any **four** questions.
Each question has weight 2.

- 25 Find the curvature of the helix $r(t) = (a \cos t)\mathbf{i} + (a \sin t)\mathbf{j} + bt\mathbf{k}$ $a, b \geq 0, a^2 + b^2 \neq 0$.
- 26 Verify Green's theorem for $\mathbf{F}(x, y) = (x - y)\mathbf{i} + x\mathbf{j}$ and the region \mathbb{R} bonded by the unit circle $r(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + 0 \leq t \leq 2\pi$.
- 27 Verify Gauss's divergence theorem for the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.
- 28 Solve the equation $\frac{dy}{dx} + y \cos x = y^n \sin 2x$.
- 29 Sketch the hyperbola $8y^2 - 2x^2 = 16$.
- 30 Find a polar equation for an ellipse with semi major axis 39.44 AU and eccentricity 0.25.

(4 × 2 = 8)

Part D

*Answer any two questions.
Each question has weight 4.*

31. Use surface integral in Stoke's theorem to calculate the circulation of the field $\mathbf{F} = x^2 \mathbf{i} + 2x\mathbf{j} + z^2 \mathbf{k}$ around the curve c , the ellipse $4x^2 + y^2 = 4$ in the xy plane, counter clockwise.
32. (a) Solve the equation :

$$xyp^2 - (x^2 + y^2)p + xy = 0$$

- (b) Solve the equation :

$$y^2 - (y - xp) = x^4 p^2$$

33. (a) Sketch the circle $r = 4 \cos \theta$. Give polar co-ordinates for the centers and identify the radius.
- (b) A wheel of radius a rolls along a horizontal straight line. Find parametric equation for the path traced by a point P on the wheel's circumference.

(2 × 4 = 8)