E 1089 (Pages : 3) Reg. No.................................

Name...................................

B.Sc. DEGREE (CBCSS) EXAMINATION, NOVEMBER 2010

First Semester

Complementary Course—Mathematics

DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course to Physics/Chemistry/Petro chemical/Geology Food Science and Quality Control/Computer Maintenance and Electronics)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
A bunch of 4 questions has weight 1.

I. 1. $\lim_{x \to 5} \frac{4}{x-7} =$ ————.

2. If $3 - x^3 \leq g(x) \leq 3\cos x$ for all x, then $\lim_{x \to 0} g(x) =$ ————.

3. The slope of the curve $y = x + \frac{2}{x}$ at $x = 1$ is ————.

4. If $y = x - \cot x$, then $\frac{dy}{dx} =$ ————.

II. 5. If $x = 5t + 1$ and $y = t^2 + 1$, then $\frac{dy}{dx}$ at $t = 5$ is ————.

6. The absolute minimum value of $f(x) = 4 - x^2, -3 \leq x \leq 1$ is ————.

7. If $f'(x) = 2x$ for all x and $f(1) = 0$, then for all x, $f(x) =$ ————.

8. Let f be a function defined on an interval I and let x_1, x_2 be any points of I. Then f is said to be increasing on I if ———— whenever $x_1 < x_2$.

III. 9. The critical points of $f(x) = x^3 - 12x + 4$ are ————.

10. Does $f(x) = x^{2/3}$, $[-1, 8]$ satisfy the hypotheses of the mean value theorem ? Justify.

Turn over

11. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 - xy + y^2$.

12. Find $\frac{\partial^2 f}{\partial x \, \partial y}$ if $f(x, y) = e^{xy}$.

IV. 13. Write the chain rule formula for $\frac{dz}{dt}$ where $z = f(x, y)$, $x = g(t)$, $y = h(t)$.

14. Write $\frac{\partial w}{\partial s}$ where $w = f(x)$, $x = g(r, s)$.

15. Define the hyperbolic cosine of y.

16. Express $\sin(iy)$ interms of a hyperbolic function.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17. Let $f(x) = x + 1$ and $\varepsilon = 0.01$. Find a $\delta > 0$ such that for all x with $0 < |x - 4| < \delta$, the inequality $|f(x) - 5| < \varepsilon$ holds.

18. Does the curve $y = x^3$ ever have a negative slope ? If so where ? Give reasons for your answer.

19. Show that $f(x) = x^3 + 3x + 1$ has exactly one zero in $[-1, 1]$.

20. Find the interval on which the function $f(x) = -x^3 + 2x^2$ is increasing.

21. Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 t}{\partial y \, \partial x}$ if $f(x, y) = x + y + xy$.

22. Use Chain rule to find $\frac{dw}{df}$, where $w = x^2 y - y^2$, $x = \sin t$, $y = e^t$.

23. Show that $\sinh(x + iy)$ has period $2\pi i$.

24. Separate $\cosh(\alpha + i\beta)$ into its real and imaginary parts.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Applying the definition of limit show that $\lim_{x \to 1}\left(\frac{3}{2}x - 1\right) = \frac{1}{2}$.

26. If $ax^2 + 2hxy + by^2 = 1$, where a, b, h are constants, use implicit differentiation to show that :

$$(ax + hy)^3 \frac{d^2y}{dx^2} = \left(ab - h^2\right)\left(hx^2 + bxy + axy + hy^2\right).$$

27. Find the absolute maximum and minimum values of $f(x) = x^{4/3}$ in $[-1, 8]$.

28. State and prove the mean value theorem.

29. Find $\frac{\partial w}{\partial u}$ as a function of u and v both by using by chain rule and by expressing w directly in terms of u and v before differentiating, given that $w = xy + yz + xz,\ x = u + v,\ y = u - v,\ z = uv$.

30. Separate $\tan^{-1}(\alpha + i\beta)$ into its real and imaginary parts.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. (a) Apply the definition of the derivative to show that derivative of a constant function is zero.
    (b) If f has a derivative at $x = c$, prove that f is continuous at c.
    (c) Show that $f(x) = |x|$ is differentiable on $(-\infty, 0)$ and $(0, \infty)$ but has no derivative at $x = 0$.

32. (a) State the mixed derivative theorem.
    (b) Find all the second order partial derivatives of the function :

$$f(x, y) = xy^2 + x^3y^3 + x^3y^4.$$

Also compute f_{xy} at $(1, 2)$.

33. Sum to infinity the series :

$$c\sin\alpha + \frac{c^2}{2}\sin 2\alpha + \frac{c^3}{3}\sin 3\alpha + \cdots.$$

$(2 \times 4 = 8)$