

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014****Fourth Semester****FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND  
ABSTRACT ALGEBRA**

(2011 Admission onwards)

[Complementary Course to Physics, Chemistry, Petrochemicals, Geology, Food Science and  
Quality Control and Computer Maintenance and Electronics]

Time : Three Hours

Maximum Weight : 25

**Part A***Answer all questions.**Each bunch of four questions has weight 1.*

- I. 1 Find the smallest positive period of  $\sin 2\pi x$ .  
2 Determine whether the function  $x|x|$  even, odd or neither even nor odd.  
3 Write the Legendre's equation.  
4 Write Bessel function of the first kind of order  $n$ .
- II. 5 Form a partial differential equation by eliminating the arbitrary constants from the equation  $z = ax + by + a$ .  
6 Write a parametric equation of the spherical surface  $x^2 + y^2 + z^2 = a^2$ .  
7 Write the Lagrange's partial differential equation.  
8 If  $F = ax^2 + by^2 + cz^2 - 1$  and  $G = x + y + z - 1$ , find  $\frac{\partial(F, G)}{\partial(y, z)}$ .
- III. 9 If  $X = 0.51$  and is correct to 2 decimal places, find  $\Delta X$ .  
10 Round off 38.46235 to four significant figures.  
11 Find the relative error of the number 8.6 if both of its digits are correct.  
12 Write the Maclaurin expansion of  $\sin x$ .
- IV. 13 Let  $*$  be defined on  $\mathbb{Q}$ , the set of rationals by letting  $a * b = ab$ . Is  $(\mathbb{Q}, *)$  a group.  
14 Find the order of the cyclic subgroup of  $\mathbb{Z}_4$  generated by 3.  
15 Define a symmetric group of  $n$  letters.  
16 Give a basis for the vector space  $\mathbb{R}(\sqrt{2})$  over  $\mathbb{R}$ .

(4 × 1 = 4)

**Turn over**

## Part B

Answer any **five** questions.  
Each question has weight 1.

17. Sketch the graph of  $f(x) = x^2$ , for  $-\pi < x < \pi$ .
18. Write the first four Legendre polynomials.
19. Derive a partial differential equation by eliminating  $f$  and  $\phi$  from the relation  

$$z = f(x + ay) + \phi(x - ay).$$
20. Find the direction cosines of the tangent at the point  $(x, y, z)$  to the conic  $ax^2 + by^2 + cz^2 = 1$ ,  
 $x + y + z = 1$ .
21. Find the quotient  $q = x/y$ , where  $x = 4.536$  and  $y = 1.32$  both  $x$  and  $y$  being correct to the digits given.
22. Write the technique involved in the false position method.
23. Find all orders of the subgroups of  $Z_6$ .
24. Let  $V$  be a vector space over a field  $F$ . Define a subspace of  $V$  over  $F$ . Also show that intersection of subspaces of  $V$  is again a subspace of  $V$  over  $F$ .

(5 × 1 = 5)

## Part C

Answer any **four** questions.  
Each question has weight 2.

25. Find the Fourier series for the function 
$$f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ \pi - x & \text{if } \pi/2 < x < 3\pi/2 \end{cases}$$
26. Define Gamma function. Show that  $\Gamma(n+1) = n!$  for  $n = 0, 1, 2, \dots$
27. Find the integral curves of  $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ .
28. Find a real root of  $x^3 - x - 1 = 0$  correct to three decimal places by false position method.
29. Use bisection method to obtain a root correct to three decimal places for  $x^3 - 18 = 0$ .
30. Prove that a subgroup of a cyclic group is cyclic.

(4 × 2 = 8)



## Part D

*Answer any two questions.  
Each question has weight 4.*

31. Find the Fourier cosine series as well as the sine series for the function  $f(x) = \pi - x$ ,  $0 < x < \pi$ .
32. Find the general solution of the differential equation  $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$ .
33. Using iteration method to find a positive root between 0 and 1 of the equation  $xe^x = 1$ .

(2 × 4 = 8)