

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2016****Second Semester****Complementary Course—OPERATIONS RESEARCH—DUALITY TRANSPORTATION AND  
ASSIGNMENT PROBLEMS**

(For B.Sc. Mathematics Model II)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

**Part A**

*Answer all questions from this part.  
Each question carries 1 mark.*

1. In the primal problem if the variable  $x_j$  is unrestricted in sign, then in the dual the constraint  $j$  of \_\_\_\_\_ type.
2. If either the primal or the dual problem has an unbounded objective function value, then what one can say about the solution of the other.
3. What is the relationship between the feasible dual and primal solutions ?
4. Define the term triangular basis.
5. Define the term optimal solution with reference to a transportation problem.
6. Define loop of a transportation table.
7. What are occupied cells ?
8. What is an unbalanced transportation problem ?
9. When does the transportation problem have a unique solution ?
10. What is the minimum number of cells in a loop ?

(10 × 1 = 10)

**Part B**

*Answer any eight questions.  
Each question carries 2 marks.*

11. Write the dual of the problem :

$$\begin{aligned} &\text{Minimise } z = 3x_1 - 2x_2 + 4x_3 \\ &\text{Subject to } \begin{aligned} 3x_1 + 5x_2 + 4x_3 &\geq 7 \\ 6x_1 + x_2 + 3x_3 &\geq 4 \\ 7x_1 - 2x_2 - x_3 &\leq 10 \\ x_1 - 2x_2 + 5x_3 &\geq 3 \end{aligned} \end{aligned}$$

Turn over

12. What are the advantages of duality ?
13. What is the significance of dual variables in a simplex solution ?
14. What is meant by non-degenerate basic feasible solution of a transportation problem ?
15. State the assignment problem.
16. What is degeneracy in transportation problem ?
17. What are the general rules of obtaining the dual of a given primal ?
18. What is meant by optimality test related to a transportation problem ?
19. Describe the assignment problem.
20. Write any three methods of solving an assignment problem.
21. State travelling salesman problem.
22. Give two applications of assignment problem.

(8 × 2 = 16)

### Part C

*Answer any six questions.  
Each question carries 4 marks.*

23. Prove that dual of the dual is the primal.
24. Prove that the value of the objective function  $f(x)$  for any feasible solution of the primal is not less than the value of the objective function  $\phi(Y)$  for any feasible solution of the dual.
25. Show graphically that the problem

$$\begin{aligned}
 &\text{Maximize } 6x_1 + 8x_2 \\
 &\text{Subject to } -x_1 + x_2 \geq 2 \\
 &\quad -5x_1 + 5x_2 \leq 3 \\
 &\quad x_1 \geq 0, x_2 \geq 0
 \end{aligned}$$

has no feasible solution.

26. Solve the transportation problem for minimum cost with the cost co-efficients, demands and supplies as given below :

|                | D <sub>1</sub> | D <sub>2</sub> | D <sub>3</sub> | D <sub>4</sub> |    |
|----------------|----------------|----------------|----------------|----------------|----|
| O <sub>1</sub> | 1              | 2              | -2             | 3              | 70 |
| O <sub>2</sub> | 2              | 4              | 0              | 1              | 38 |
| O <sub>3</sub> | 1              | 2              | -2             | 5              | 32 |
|                | 40             | 28             | 30             | 42             |    |

27. Explain how transportation problem is solved when supply and demand are not equal.
28. A company has factories A, B and C which supply warehouses at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90, 110 and 160 units respectively. Unit shipping costs in rupees are as follows :

|   | D  | E  | F  | G  |
|---|----|----|----|----|
| A | 42 | 48 | 38 | 37 |
| B | 40 | 49 | 52 | 51 |
| C | 39 | 38 | 40 | 43 |

Determine the optimum distribution for this company to maximise shipping costs.

29. Explain the transshipment problem.
30. Summarize the computational algorithm used for solving transportation problems.
31. How would you deal with the assignment problems where
- The objective function is to be maximised ;
  - Some assignments are prohibited.

(6 × 4 = 24)

#### Part D

Answer any **two** questions.

Each question carries 15 marks.

32. Solve by revised simplex method :

$$\text{Minimise } -5x_1 + x_2 - x_3 + 10x_4 - 7x_5$$

$$\text{Subject to } \begin{bmatrix} 3 & -1 & -1 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 2 & 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}, x \geq 0$$

Turn over



33. A batch of four jobs can be assigned to five different machines. The set up time for each job on each machine is given below. Find an optimal assignment of jobs to machines which will minimize the total set up time.

|      |   | Machines |    |    |    |    |
|------|---|----------|----|----|----|----|
|      |   | 1        | 2  | 3  | 4  | 5  |
| Jobs | 1 | 10       | 11 | 4  | 2  | 8  |
|      | 2 | 7        | 11 | 10 | 14 | 12 |
|      | 3 | 5        | 6  | 9  | 12 | 14 |
|      | 4 | 13       | 15 | 11 | 10 | 7  |

34. A caterer needs clean table covers every day for six days to meet a contract according to the following table :

|               |   |    |    |    |    |    |     |
|---------------|---|----|----|----|----|----|-----|
| Days          | : | 1  | 2  | 3  | 4  | 5  | 6   |
| No. of Covers | : | 50 | 60 | 80 | 70 | 90 | 100 |

The cost of a new cover is Rs. 20 while washing charges are Re. 1 for return on the fourth day or later Rs. 2 for return on the third day and Rs. 3 for the next day. Find the minimum cost schedule for the purchase and washing of table covers, assuming that after the end of the contract the covers are rejected.

35. A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in Rs. '000) of each city from a particular city is given below :

|   | A  | B | C | D | E |
|---|----|---|---|---|---|
| A | —  | 2 | 5 | 7 | 1 |
| B | 6  | — | 3 | 8 | 2 |
| C | 8  | 7 | — | 4 | 7 |
| D | 12 | 4 | 6 | — | 5 |
| E | 1  | 3 | 2 | 8 | — |

What is the sequence of visit of the salesman, so that the cost is minimum ?

$$(2 \times 15 = 30)$$