QP CODE: 24018062

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2024

Fourth Semester

Elective - ME800402 - ALGORITHMIC GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

EB6D5BAE

Time: 3 Hours Weightage: 30

*Instructions: (**Applicable for Private Registration, 2020 Admission Onwards**) This question paper contains two sections. Answer section I questions in the answer book provided. Section II Internal examination questions must be answered in the question paper itself. Follow the detailed instructions given under section II.*

SECTION I

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that $\bar{G}$ is regular if and only if G is regular.
2. Define a unilateral digraph. Give an example of a unilateral digraph that does not contain a cycle.
3. Define floor and ceiling of a number with examples.
4. Define an $m-ary$ tree. Give an example
5. Define the distance between two vertices of a graph G. Describe with examples.
6. Define center and median of a graph G.
7. Define value of a flow f in a network N.
8. Define edge connectivity and vertex connectivity of a graph. Give an example of a graph with $\kappa(G) = \lambda(G) = \delta(G)$
9. *Define a feasible vertex labeling of a weighted complete bipartite graph*

10. *Define a decomposition of a graph G.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define degree set of a graph. What is the degree set of an $r-$regular graph. Draw a graph G having the degree set $\mathcal{D}(G) = \{0, 4, 5\}$.

12. Draw the digraph D with $V(D) = \{v_1, v_2, v_3, v_4, v_5\}$, $A(D) = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_1, v_4), (v_2, v_5), (v_4, v_5), (v_3, v_4)\}$. Determine its adjacency matrix, adjacency list and adjacency list table.

13. Prove that a tree of order p has size $p - 1$.

14. Prove that Prim's algorithm produces a minimum spanning tree in a non - trivial connected weighted graph

15. State and prove the Max-Flow Min-Cut Theorem.

16. Prove that for $n \geq 1$, a graph G is $n-$ connected. if and only if every pair of vertices of G is connected by at least $n-$ internally disjoint paths.

17. *Define an underlying graph of a multigraph. Prove that every r$-$regular bipartite multigraph, r ≥ 1 has a perfect matching.*

18. Define a BIBD. Show that there is no BIBD with $b = v = 46$; $r = k = 10$ and $\lambda = 2$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Show that every $u - v$ walk in a graph contains a $u - v$ path.
    b) An edge e of a connected graph is a bridge if and only if e does not lie on a cycle of G.

20. Explain DFS Algorithm using an example. Find its complexity.

21. Let N be a network and f a flow in N. If $f(P, \overline{P})$ is a cut of N then prove that the value of the flow in N is given by $f(N) = f(P, \overline{P}) - f(\overline{P}, P)$

22. *a) Define a Hamiltonian graph with example*
    b) Prove that for every positive integer n , the graph K_{2n+1} can be factored into n Hamiltonian cycles
    c) Give an ascending subgraph decomposition of the Petersen graph

(2×5=10 weightage)