G 3502 (Pages : 2) Reg. No...............................

Name...............................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015

Second Semester

Faculty of Science

Branch I (a)—Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Why Urysohn characterisation of normality theorem is remarkable ?
2. Define T_4 spaces. When they are Tychonoff ? Explain.
3. Explain : Coproduct and its summands.
4. Define cantor discontinuum.
5. What do you mean by "Filter associated with a net" ?
6. Explain the notion of sub-bases for filters.
7. State *four* equivalent condition for a T_1 topological space X to be countably compact.
8. State two results about compactness that continue to hold for countable compactness.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each questions has weight 2.

9. Let C_i be a closed subset of a space X_i for $i \in I$ show that $\prod_{i \in I} C_i$ is a closed subset of $\prod_{i \in I} X_i$ w.r. to product topology.
10. Justify the terms 'box' and 'wall' geometrically for products of copies of the red-line.
11. Characterise evaluation functions.
12. Prove : A topological space is compact iff every ultrafilter in it is convergent.

Turn over

13. Let X be a set and Y be any singleton set. Prove that $\Im$ is an ultrafilter on X iff there exists function $\theta : P(X) \to P(Y)$ preserving intersections and complements such that $\Im = \theta^{-1}(\{Y\})$.

14. Explain :

(i) Filter base. (ii) $\Im$-neighbourhood filter.

(iii) Cluster point of a filter. (iv) Image filter.

(v) Sub-filter.

15. Prove that every locally compact Hausdorff space is regular. Explain that it is not the best possible result.

16. Explain : Sequential compactness is not a productive property but it is finitely productive.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove Tietze characterisation of normality theorem.

18. Characterise (a) Completely regular topological space ; (b) Regular topological product ; and (c) One-to-one evaluation function.

19. State and prove Urysohn metrisation theorem.

20. (a) Characterise continuity in terms of convergence of filters.

(b) Characterise convergence of filters in topological products.

21. Establish : There is a canonical way of :

(a) Passing from nets to filters and vice-versa.

(b) Bring out the relationship between filter and ultra filter.

22. Prove :

(a) Local compactness is preserved under continuous, open functions and that it is a weakly hereditary property.

(b) One-point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff.

$(3 \times 5 = 15)$