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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2022

First Semester

CORE - ME010102 - LINEAR ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

5B7BFB34

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers. Define (x, y) + (x_1, y_1) = (x + x_1, y + y_1) and *c(x, y) = (cx, y). Is V, with these operations, a vector space over the field of real numbers?*
2. Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let V_e be the subset of even functions and let V_o be the subset of odd functions. Then prove that V_e and V_o are subspaces of V.
3. Define a linear transformation and prove that the differential operator D on the vector space V of all polynomial functions over a field F is linear.
4. If $T : C^2 \to C^2$ is a linear operator defined as $T(x_1, x_2) = (x_1, 0)$, find the matrix of T in the ordered basis $\{(1,i),(-i,2)\}$ for C^2 .
5. Let V be a finite dimensional vector space over F. Prove that the mapping $C : V \to V^{**}$ by $C\alpha = L_\alpha$ where $L_\alpha(f) = f(\alpha); \forall f \in V^*$ is linear.
6. Show that the function D from K^{nxn} to K defined by *D(A)=a A(1, k_1)...A(n, k_n) is n- linear, where a $\in$K and k_1,...,k_n are positive integers less than or equal to n.*
7. If D is a 2-linear function with the property that D(A) = 0 for all 2 x 2 matrices A over K having equal rows, then show that D is alternating.
8. *If A is an invertible n x n matrix over a field, show that det A ≠ 0.*
9. *Define minimal polynomial for a linear operator on a finite dimensional vector space* V *. What is the minimal polynomial for the identity operator on* V *?*
10. *Prove that every matrix* A *such that* $A^2 = A$ *is similar to a diagonal matrix.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *If W is a proper subspace of a finite-dimensional vector space V, then prove that W is finite-dimensional and dim W < dim V.*

12. *Show that the vectors α_1 = (-1, 0, 0), α_2 = (4, 2, 0), α_3 = (5, 3, -8) form a basis for* $\mathbb{R}^3$ *. Find the coordinates of the vector (1, 2, 1) in the ordered basis {α_1, α_2, α_3}.*

13. Show that if F is any field then $F^{m\times n}$ is isomorphic to F^{mn} .

14. Find the linear functional f on R^3 such that $f(1,0,1)=1,\ f(0,1,-2)=-1,\ f(-1,-1,0)=3.$

15. Let V and W be finite dimensional vector spaces over the field F and $T:V\rightarrow W$ is a linear transformation. Prove that $Rank(T^t)=Rank(T).$

16. *Show that det (AB) = (det A) (det B), if A and B are two n x n matrices over K.*

17. Find the characteristic values *and characteristic vectors* of the linear operator T on $\mathbb{C}^2$ which is represented in the standard ordered basis by the matrix $A=\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$

18. *Let T be a linear operator on a finite dimensional vector space V. Let $c_1, c_2, \cdots, c_k$ are the distinct characteristic values of T and W_i 's are the corresponding characteristic spaces. Then prove that T is diagonalizable if and only if* $\dim W_1+\dim W_2+\cdots+\dim W_k=\dim V$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. *Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and dim W $\leq$ m. Then prove that there is precisely one m x n row-reduced echelon matrix over F which has W as its row space.*

20. Let V and W be finite dimensional vector spaces over the field F. By exhibiting a basis for the space $L(V,W)$,prove that $L(V,W)$ is finite dimensional and $dimL(V,W)=dimV.dimW.$

21. (a)Let K be a commutative ring with identity and n be a positive integer. Let A be an n x n matrix over K. Then show that A is invertible over K if and only if det A is invertible in K. When A is invertible, show that the unique inverse for A is A^{-1} =($det\ A)^{-1}$ adj A.

(b) *Let K=$\mathbb{R}[x]$, the ring of polynomials over $\mathbb{R}$. Check whether the matrices A=* $\begin{bmatrix}x^2+x & x+1\\ x-1 & 1\end{bmatrix}$ *and B=* $\begin{bmatrix}x^2-1 & x+2\\ x^2-2x+3 & x\end{bmatrix}$ *are invertible over K and if invertible, find the inverse.*

22. Let $V=W_1\oplus\cdots\oplus W_k$, prove that there exist k linear operators $E_1, E_2, \cdots, E_k$ on V such that

    i. Each E_i is a projection
    ii. $E_iE_j=0$ if $i\neq j$
    iii. $I=E_1+\cdots+E_k$
    iv. The range of E_i is W_i

Also prove that if $E_1, E_2, \cdots, E_k$ are k linear operators on V which satisfy conditions (i), (ii) and (iii) and if we let W_i be the range of E_i then $V=W_1\oplus\cdots\oplus W_k$

(2×5=10 weightage)