

G 18001484



18001484

Reg. No.....

Name.....

**M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2018**

**Second Semester**

Faculty of Science

Branch I (A) : Mathematics

MT 02 C08—ADVANCED COMPLEX ANALYSIS

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

**Part A**

*Answer any **five** questions.*

*Each question has weight 1.*

1. State Morera's theorem.
2. Find the radius of convergence of  $\sum \frac{n!}{n^n} \cdot z^n$ .
3. State Arzela's theorem.
4. Define genus of  $f(z)$ . If genus is zero. Write down the product development of  $f(z)$ .
5. Explain (i) simply connected region ; (ii) conformal mapping.
6. Give an example of subharmonic function and justify your answer.
7. Define meromorphic function with an example.
8. Explain the concept of lifting an arc.

(5 × 1 = 5)

**Part B**

*Answer any **five** questions.*

*Each question has weight 2.*

9. Give an example to show that the convergence of  $\sum_1^{\infty} a_n$  is neither sufficient nor necessary for the convergence of the product  $\sum_1^{\infty} (1 + a_n)$ .

**Turn over**





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10. State and prove Hurwitz theorem.
11. Characterise a totally bounded family of function.
12. Obtain a characterization of normal family of functions.
13. Explain : (i) Real analytic arc ; (ii) regular analytic arc ; (iii) free boundary arc ; and (iv)  $Z$  approaches the boundary of  $\Omega$ .
14. State Schwarz-Christoffel formula and its another version.
15. State and prove the property on the number of zeros and poles of an elliptic function.
16. What is the homotopy group of a disk ? Prove.

(5 × 2 = 10)

### Part C

*Answer any **three** questions.  
Each question has weight 5.*

17. Prove :

(i)  $\sqrt{z+1} = z\sqrt{z}$  .

(ii)  $\sqrt{z}\sqrt{1-z} = \pi/\sin \pi z$  and

(iii) Obtain Legendre's duplication formula.

18. Derive the functional equation and obtain its another form.
19. (i) Obtain sufficient condition for a function to be harmonic.  
(ii) List the properties of subharmonic function.
20. Obtain Harnack's inequality and deduce Harnack's principle.
21. Obtain the addition theorem for Weierstrass  $\wp$  function.
22. List and prove the general properties of elliptic functions.

(3 × 5 = 15)

