G 2794 (Pages : 3) Reg. No.................................

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# M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C 10—REAL ANALYSIS

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

## Part A

*Answer any* **five** *questions.*
*Each question has weight* 1.

1. Boundedness of $f'$ is not necessary for $f$ to be of bounded variation. Explain with example.
2. Define oriented circle and write its equation.
3. Show that $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$.
4. Define unit step function and sketch it.
5. State the Stone–Weierstrass theorem.
6. What is the difference between uniform convergence and pointwise convergence of sequences ?
7. Show that the function $e^x$ is strictly increasing.
8. Evaluate $\int_{-\pi}^{\pi} e^{inx} dx$ for $n \in Z$.

$(5 \times 1 = 5)$

## Part B

*Answer any* **five** *questions.*
*Each question has weight* 2.

9. Define function of bounded variation with example. Prove your example.
10. State the theorem on characterisation of rectifiable curves.
11. If $f \in \mathscr{R}(\alpha)$ prove $|f| \in \mathscr{R}(\alpha)$.
12. If $p^*$ is a refinement of $p$ prove : $L(p,f,\alpha) \leq L(p^*,f,\alpha)$ and $U(p^*,f,\alpha) \leq U(p,f,\alpha)$.

Turn over

13. Establish Cauchy criterion for uniform convergence of sequence of functions.

14. Give an example of an everywhere discontinuous limit function, which is not Riemann-integrable.

15. Give example to show that :

$$\sum_i \sum_j a_{ij} \neq \sum_j \sum_i a_{ij} \text{ and prove.}$$

16. Evaluate :

$$\lim_{x \to 0} \frac{e-(1+x)^{1/x}}{x}.$$

(5 × 2 = 10)

## Part C

*Answer any* **three** *questions.*
*Each question has weight 5.*

17. (a) State and prove the theorems on properties of arc length.

 (b) Explain equivalence of path with illustrations.

18. (a) State and prove the theorem to show that integration and differentiation are, in a sense, inverse operations.

 (b) State and prove theorem on integration by parts.

19. (a) State and prove change of variables theorem on Riemann-Stieltjes integral.

 (b) If $f$ is continuous on $[a, b]$ prove $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

20. (a) State the theorem to prove :

$$\int_a^b f d\alpha = \lim_{n \to \infty} \int_a^b f_n d\alpha \text{ if } f_n \to f \text{ uniformly on } [a, b).$$

 (b) Prove the existence of a real continuous function on the real line which is nowhere differentiable.

21. (a) State the theorem to establish :

$$\lim_{t \to x} \lim_{n \to \infty} f_n(t) = \lim_{n \to \infty} \lim_{t \to x} f_n(t).$$

 (b) Discuss the main problem of the sequence of functions.

22. If $f(x) = (\pi - |x|)^2$ on $[-\pi, \pi]$.

Prove :

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx.$$

Deduce :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \text{ and } \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

(3 × 5 = 15)