G 3504 (Pages : 2) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015

Second Semester

Faculty of Science

Branch : I (A) Mathematics

MT 02 C 09—PARTIAL DIFFERENTIAL EQUATIONS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Find the general integral of $Z_t + ZZ_x = 0$.
2. Explain the classification of integrals.
3. Find the complete integral of $zpq - p - q = 0$.
4. Write down the characteristic equation of the non-linear equation $F(x, y, z, p, q) = 0$.
5. Show that $z = \frac{1}{x}\phi(y-x) + \phi^{1}(y-x)$ satisfies $\partial^2 z/\partial x^2 - \partial^2 z/\partial y^2 = 2z/x^2$.
6. Explain the origin of second order equation.
7. Determine the region where $u_{xx} - 2x^2 u_{zz} + u_{yy} + u_{zz} = 0$ is of hyperbolic.
8. Explain : Dirichlet problem of interior type.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. **Find the general integral of $2x\left(y + z^2\right) p + y\left(2y + z^2\right) q = z^3$.**
10. **Find the orthogonal trejectiories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of plane parallel to $z = 0$.**
11. **By method of characteristics, find the integral surface of $pq = xy$ which passes through the curve $z = x, y = 0$.**

Turn over

12. Show that $f = xp - yq - x = 0$ and $g = x^2p + q - xz = 0$ are compatible and find a one parameter family of common solution.

13. Solve $r + s - t = e^{x+y}$.

14. Reduce to canonical form $u_{xx} - x^2 u_{yy} = 0$.

15. Obtain the condition for the surfaces $f(x, y, z) = c$ to form an equipotential family of surfaces and find its general form of the potential function.

16. By method of separation of variable show that $\frac{\partial^2 z}{\partial x^2} = \frac{1}{e^2}\frac{\partial^2 z}{\partial t^2}$ has solution of the form $A \exp\left(\pm inx \pm inct\right)$ where A and B are constans.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. State and prove the method of finding a general integral of a quasi linear equation.

18. Obtain memory and sufficient condition for the integrability of $dz = Q\left(x, y, z\right) dx + \psi\left(x, y, z\right) dy$.

19. Obtain two complete integrals of $Z^2 (1 + p^2 + q^2) = 1$. Are they equivalent ? Why ?

20. Explain Jacobi's method. Use it to find the solution of $u_z\, x^2 - u_y^2 - a\, u_z^2 = 0$.

21. Find the solution of $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x + y}$ Valid when $x, y > 0$, $xy > 1$ such that $z = 0$, $p = \frac{2y}{x + y}$ on the hyperbola $xy = 1$.

22. Reduce :

    (a) $\sin^2 x\, u_{xx} + 2 \cos x\, u_{xy} - u_{yy} = 0$ to canonical form.

    (b) Describe the method of separation of variables.

(3 × 5 = 15)