F 5113 (Pages : 2) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY/FEBRUARY 2017

First Semester

Faculty of Science

Branch I (a)—Mathematics

MT 01 C04—GRAPH THEORY

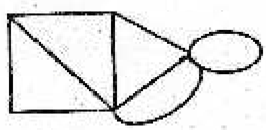
(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries **1** *weight.*

1. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.
2. Show that if a simple graph G is not connected, then G^c is connected.
3. Show that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
4. Determine the number of spanning trees of the following graph :



5. Prove that a subset S of V is independent if and only if $V \mid S$ is a covering of G.
6. Does there exists an Eulerian graph with an odd number of vertices and on even number of edges. Draw such a graph if it exists.
7. Prove that a graph is planar if and only if it is embeddable on a sphere.
8. Prove that for any simple planar graph $G \cdot \delta(G) \leq 5$.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries **2** *weight.*

9. Prove that every tournament contains a directed Hamiltonian path.
10. Prove that in a 2-connected graph G, any two longest cycles have atleast two vertices in common.

Turn over

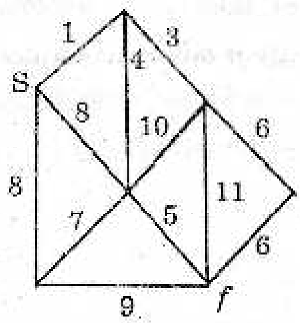
11. Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.
12. Describe Prim's algorithm.
13. State and prove Ore's theorem.
14. Prove that in a critical graph G, no vertex cut is a clique.
15. If G is a loopless bipartite graph, show that X' (G) = A (G).
16. Prove that K_5 is non-planar.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries 5 *weight.*

17. (a) Prove that a graph is bipartite if and only if it contains no add cycles.
    (b) Prove that in a connected graph G with at least three vertices, any two longest paths have a vertex in common.
18. (a) Describe Dijkstra's algorithm.
    (b) Determine a minimum-weight *s-f* path using Dijkstra's algorithm for the graph given below :



19. (a) Prove that a connected graph G with atleast two vertices is a tree if and only if it degree sequence $(d_1, d_2 \ldots d_n)$ satisfies the condition $\sum_{i-1}^{n} d_i = 2(n-1)$ with $d_i > 0$ for each i.
    (b) Prove that every 3-edge connected graph has three spanning trees with empty intersection.
20. For a connected graph G prove that the following statements are equivalent :
    (i) G is Eulerian.
    (ii) The degree of each vertex of G is an even positive integer.
    (iii) G is an edge-disjoint union of cycles.
21. Prove that if a connected graph G is neither an odd cycle nor a complete graph then X (G) ≤ A (G).
22. State and prove Heawood five-color theorem.

(3 × 5 = 15)