G 1992

(Pages : 2)

Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MARCH 2015

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT 01 C02—BASIC TOPOLOGY

(2012 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Differentiate between :
   (a) Finer and coarser topologies.
   (b) Discrete and indiscrete topologies.
2. Define closed set and open set. Also give examples of sets :
   (a) Both closed and open.
   (b) Neither closed nor open.
3. Show that Composition of continuous functions is also continuous.
4. Define homeomorphism with an example. Also prove your result.
5. Explain the concepts of separated sets, mutually disjoint sets and connected sets.
6. Prove : Components of open subsets of a locally connected space are open.
7. Prove that compact subsets in Hausdorff space are closed.
8. Verify that a discrete space satisfies all separation axioms.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that if a space is second countable then every open cover of it has a countable sub-cover.
10. Illustrate pictorially : Metric topology on $\mathbb{R}^2$ is same as the product topology.
11. Explain extension problem and its dual.

Turn over

12. Prove that every second countable space is first countable but not conversely.

13. Show that the finite Cartesian product of connected space is also connected.

14. Differentiate between connectedness and locally connectedness with examples.

15. Show that a subspace of a completely regular space is completely regular. Also show that a product of completely regular spaces is completely regular.

16. Prove that all metric spaces are T_4.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Establish the existence of infima in the partially ordered set of all topologies on a set X.

    (b) Characterize subbases.

18. (a) Establish equivalent conditions for f to be continuous at x_0.

    (b) State and prove Lebesgue Covering lemma.

19. (a) Use Lebesgue Covering lemma to prove that every continuous function from a compact metric space into another metric space is uniformly continuous.

    (b) Define weakly hereditary property with two examples and prove your result.

20. (a) Establish four equivalent conditions for a space to be connected.

    (b) Show that the unit sphere in R^{n+1} is connected.

21. (a) Show that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger conditions.

    (b) Prove that in a Hausdorff space limits of sequences are unique.

22. Prove that every regular Lindeloff space is normal.

$(5 \times 3 = 15)$