QP CODE: 21002037

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Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2021

First Semester

CORE - ME010105 - GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

FB9900E7

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that the order of a self complementary graph is either $4k$ or $4k+1$ where k is a positive integer.
2. If the bipartite graph $G(X,Y)$ is regular, show that $|X| = |Y|$
3. Show that no vertex v of a simple graph can be a cut vertex of both G and G^c.
4. (a) Define edge conraction in graphs and write the formula for finding the number of spanning trees in a connected labelled graph.
   (b) Find $\tau(C_4)$
5. Write a short note on any two particular cases of the connector problem.
6. Does there exists an Eulerian graph with (i) an even number of vertices and an odd number of edges and (ii) an odd number of vertices and an even number of edges. Draw such a graph if it exists.
7. Determine the chromatic number of the Petersen graph.
8. For any graph G, prove that $\chi(G) \leq 1 + \Delta(G)$.
9. *What you mean by the faces of a planar graph.*
10. *Show that a planar graph with minimum degree at least 5 contains at least 12 vertices.*

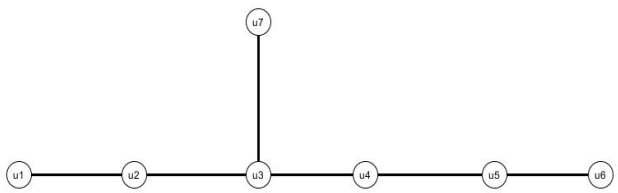
(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that the following graph is an identity graph



12. Find the order and size of $G_1 \times G_2$

13. Show that a graph G with at least two vertices is 2-connected if and only if any two edges of G lie on a common cycle.

14. State and prove Ear decomposition theorem of a block.

15. Define closure of a graph. Describe the construction of the closure of a graph G. Give an example.

16. For any simple graph G, prove that $2\sqrt{n} \leq \chi + \chi^c \leq n + 1$ and $n \leq \chi\chi^c \leq ((n+1)/2)^2$

17. *Show that K_5 is a nonplanar graph using Jordan Curve Theorem*

18. *What is the spectrum of C_n?. Explain.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19.
   a. State and prove Moon's theorem.
   b. State and prove Redei's theorem.

20.
   a. Show that every tree has a centre consisting of either a single vertex or two adjacent vertices.
   b. Prove that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path.
   c. Show that a connected graph G is a tree if and only every edge of G is a cut edge.

21. a. Prove that if G is a simple graph with $n \geq 3$ and $\delta \geq \frac{n}{2}$ then G is hamiltonian.
   b. Prove that the converse of the statement in part a is not true in general.
   c. Prove that the wheel graph W_n is hamiltonian for every $n \geq 4$

22. *Prove that every planar graph is 5 – vertex colorable.*

(2×5=10 weightage)