QP CODE: 24018058

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2024

Fourth Semester

Core - ME010402 - ANALYTIC NUMBER THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

F8DFAACE

Time: 3 Hours | Weightage: 30

Instructions (Applicable for **Private Registration,2020 Admission Onwards**) : This question paper contains two sections. Answer section I questions in the answer book provided. Section II Internal examination questions must be answered in the question paper itself. Follow the detailed instructions given under section II.

SECTION I

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that the $M\ddot{o}bius$ function is multiplicative but not completely multiplicative.

2. If α has a Dirichlet inverse α^{-1} , then prove that the equation $G(x) = \sum_{n\leq x} \alpha(n)F(\frac{x}{n})$ implies $F(x) = \sum_{n\leq x} \alpha^{-1}(n)G(\frac{x}{n})$ and conversely.

3. Write an asymptotic formula for $\sum_{n\leq x} d(n)$and hence find its average order.

4. State any four relations which are logically equivalent to the prime number theorem.

5. State Shapiro's Taubarian Theorem.

6. (a)If $c > 0$ then prove that $a \equiv b(mod\ m)$ if and only if $ac \equiv bc(mod\ mc)$.
   (b) Assume $a \equiv b(mod\ m)$. If $d \mid m$ and $d \mid a$ then prove that $d \mid b$

7. If $(a, m) = 1$ then show that the solution of the linear congruence $ax \equiv b(mod\ m)$ is given by $x \equiv ba^{\phi(m)-1}(mod\ m)$.

8. State and prove wilson's theorem.

9. Prove that $(-1|p) = 1$ if $p \equiv 1(mod4)$. Also write a formula for$(2|p)$when p is an odd prime.

10. (a) Define the exponent of a modulo m .
(b)Let $m \geq 1$ and $(a, m) = 1$. Prove that the numbers $1, a, a^2, \ldots, a^{f-1}$are incongruent mod m, where, $f = exp_m(a)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. (a) Define Dirichlet product of two arithmetical functions f and g .
(b) Prove that $f * g = g * f$ and $(f * g) * k = f * (g * k)$ for any arthmetical functions $f, g \ and \ h$.

12. (a)For $x \geq 1$, prove that $\sum_{n \leq x} \Lambda(n)[\frac{x}{n}] = log[x]!$.
(b)State and prove the Legendre's identity.

13. State and prove Abel's identity.

14. Prove that there is a constan t A such that $\sum_{p \leq x} \frac{1}{p} = log \ log \ x + A + O(\frac{1}{log \ x}), \forall \ x \geq 2$.

15. (a) Assume (a,m)=1, show that the linear congruence $ax \equiv b(modm)$ has exactly one solution .
(b) Give an example of a linear congruence having no solution .

16. Prove that the set of lattice points in the plane visible from the origin contains arbitrarily large square gaps.

17. State and prove Euler's criterion.

18. If $(a, m) = 1$ then prove that $exp_m(a^k) = \frac{exp_m(a)}{(k,f)}$, where $f = exp_m(a)$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) For $x > 1$, prove that $\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \ log \ x)$.

(b) Two lattice points (a, b) and (m, n) are mutually visible if and only if $a - m$and $b - n$are relatively prime.

20. State and prove the inequality showing $\frac{n}{log \ n}$ is the correct order of magnitude of $\pi(n)$.

21. (a) State and prove Chinese Remainder Theorem .

(b) $m_1, m_2, .. m_r$ are relatevely prime in pairs and $b_1, b_2, \ldots b_r$ are are arbitrary integers and let $a_1, a_2, \ldots a_r$ satisfy $(a_k, m_k) = 1\ for\ k = 1, 2, .. r$. Prove that the system of congruences $a_1 x \equiv b_1 (mod\, m_1),\ \ a_2 x \equiv b_2 (mod\, m_2), \ldots .. a_r x \equiv b_r (mod\, m_r)$ has exactly one solution $modulo\, m_1 m_2 \ldots . m_r$.

22. State and prove Gauss' lemma.

(2×5=10 weightage)