



QP CODE: 23002638



23002638

Reg No :

Name :

M Sc DEGREE (CSS) EXAMINATION, MARCH 2023

Third Semester

Faculty of Science

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

92C16E87

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Define orthogonal trajectories on the surface of a given system of curves.
2. Verify that the equation $yz dx + xz dy + xy dz = 0$ is integrable.
3. Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$.
4. Explain the different types of integrals of a nonlinear partial differential equations of first order.
5. Prove that the equations $p = P(x, y)$, $q = Q(x, y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.
6. If $z = f(x + iy) + g(x - iy)$ where f and g are arbitrary functions, then show that $r + t = 0$.
7. Find the particular integral of $[D^2 - D']z = e^{x+y}$.
8. The PDE $yu_{xx} + xu_{yy} = 0$ is hyperbolic in which quadrants?
9. Define a family of equipotential surfaces and corresponding potential function.
10. Show that that $\psi = \frac{q}{|r-r'|}$ where q is a constant and r and r' are the position vectors of the points (x, y, z) and (x', y', z') , is a solution of the Laplace's equation $\nabla^2 \psi = 0$

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Find the integral curves of $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$





12. Eliminate the arbitrary function f from the given equations.
 a) $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
 b) $z = x + y + f(xy)$
13. Find a complete integral of the equation $p^2x + q^2y = z$.
14. Show that the integral surface of the equation $z(1 - q^2) = 2(px + qy)$ which passes through the line $x = 1, y = hz + k$ has an equation $(y - kx)^2 = z^2\{(1 + h^2)x - 1\}$.
15. Solve $[D^3 - 2D^2D' - DD'^2 + 2D'^3]z = 0$.
16. Solve $r - s + 2q - z = x^2y^2$.
17. Prove that if $f(x, y, z) = c$ is a family of equipotential surfaces, then $\frac{\nabla^2 f}{|\nabla f|^2}$ is a function of f alone.
18. Prove that the function $\phi = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ is Harmonic and find the corresponding analytic function $\phi + i\psi$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Prove the following.
 a) A necessary and sufficient condition that the Pfaffian differential equation $X \cdot dr = 0$ should be integrable is that $X \cdot \text{curl} X = 0$.
 b) Given one integrating factor of the Pfaffian differential equation $X_1 dx_1 + X_2 dx_2 + \dots + X_n dx_n = 0$, we can find an infinity of them.
20. Deduce that the general solution of the linear partial differential equation $Pp + Qq = R$ is of the form $F(u, v) = 0$ where F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Hence determine the general solution of the partial differential equation $z(xp - yq) = y^2 - x^2$.
21. By Jacobi's method, solve $z^2 + zu_z - p^2 - q^2 = 0$.
22. Solve the one- dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}$ by separating the variables

(2×5=10 weightage)

