**QP CODE: 22002315**

22002315

**Reg No** : .....................

**Name** : .....................

# MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2022

## Second Semester

## CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

FFEEF25D

Time: 3 Hours | Weightage: 30

### Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Give an example to show that a linear transformation need not be commutative.
2. Reflect the imaginary axis in the circle $|z-2|=1$.
3. What do you mean by rectifiable arcs?
4. State Cauchy's theorem for a disk with exceptional points.
5. State Cauchy's representation formula.
6. Prove that a function which is analytic in the whole plane and satisfies the inequality $|f(z)|<|z|^n$ for some n and for sufficiently large $|z|$ reduces to a polynomial.
7. Prove that the zeros of an analytic function are isolated.
8. State the local mapping theorem. Use it to prove that $\int_\gamma \frac{2z+1}{z^2+z-6}dz=0$ where $\gamma$ is the unit circle.
9. If $z=a$ is a pole of order $n$ for $f(z)$ then give a formula for finding its residue.
10. State the generalized argument principle.

(8×1=8 weightage)

### Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. State and prove Cauchy criterion for convergence of a sequence.
12. Prove that an analytic function in a region $\Omega$ whose derivative vanishes identically must reduce to a constant. Also prove that the same is true if its modulus is a constant.

13. If f(z) is analytic and satisfies the inequality |f(z)-1| <1 in a region $\Omega$, then show that $\int_\gamma \frac{f'(z)}{f(z)} \mathrm{d}z$ =0.

14. Prove that the index is constant in each of the regions determined by a closed curve $\gamma$.

15. Define the algebraic order of a meromorphic function $f(z)$ at $z = a$ . Prove that the order is positive for a pole and is negative for a zero of $f(z)$ .

16. Let $f(z)$ be analytic in a region $\Omega$ and $a \in \Omega$ such that $|f(a)| \leq |f(z)|$ for every $z \in \Omega$ then prove that either $f(a) = 0$ or $f(z)$ is a constant.

17. Prove that a region $\Omega$ is simply connected iff $n(\gamma, a) = 0$ for all cycles $\gamma$ in $\Omega$ and for all points $a$ in $\Omega^C$

18. Evaluate $\int_0^\infty \frac{cosxdx}{x^2+a^2}, a > 0.$

(6×2=12 weightage)

**Part C (Essay Type Questions)**

*Answer any **two** questions.*

*Weight **5** each.*

19. (i)Prove that any circle on the sphere corresponds to a circle or a straight line in the complex plane.
(ii) Find the correspondence between the coordinates of a point on the Riemann sphere and its image in the complex plane.

20. State and prove Cauchy's theorem for a rectangle.

21. (a)If $f(z)$ is analytic in a region $\Omega$ containing the point $a$, prove that it is possible to write $f(z) = f(a) + \frac{f'(a)}{1!}(z-a) + \frac{f''(a)}{2!}(z-a)^2 ... + \frac{f^{n-1}(a)}{(n-1)!}(z-a)^{n-1} + f_n(z)(z-a)^n$ , where $f_n(z)$ is analytic in $\Omega$ .
(b) Derive the integral expression for $f_n(z)$ .

22. *If $pdx + qdy$ is locally exact differential in a region $\Omega$ , then $\int_\gamma pdx + qdy = 0$ for every curve $\gamma \sim 0(mod\Omega)$.*

(2×5=10 weightage)