G 2791 (Pages : 2) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Explain (a) Vrysohn function ; (b) Tychonoff spaces.
2. Justify : Topologically the coproducts are not as interesting as the products.
3. Define : Product topology.
4. Justify the term : Evaluation.
5. Explain the terms :

   (a) Atomics filter ; (b) Cofinite filter.
6. Prove that a function that preserves finite intersections and complements is monotonic.
7. Every continuous, real-values function on a countably compact space is bounded and attains its extrema-Prove.
8. Give an example of a locally compact Hausdorff space which is not normal. Justify your example.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Prove uniform convergence implies Pointwise convergence. Show that converse is false.
10. Prove that every metric space is perfectly normal.
11. Obtain necessary and sufficient condition for the evaluation function to be continuous.
12. Characterise those families of sets which can be bases for filter.
13. Show that every filter has the finite intersection property but the converse is not true.

Turn over

14. Obtain sufficient condition for a :
    - (a) Family to be a base for a unique filter.
    - (b) Explain the notion of sub-base for filters.
15. Prove that a space is Tychonoff iff it can be embedded into a compact Hansdorff space.
16. Justify :
    - (a) Local compactness is not a hereditary property.
    - (b) Locally compact Hansdorff space need not be normal.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. State and prove the lemmas to reduce the problem of finding a continuous function on a space to the problem of constructing a family $\{F_t : t \in Q\}$ of subsets with certain conditions.
18. State are prove Konig's theorem and show that it is equivalent to the axiom of choice.
19. State and prove the Urysohn netrisation theorem.
20. Give *two* proofs : A topological space is :
    - (a) Hansdorff iff no filter can converge to more than one point in it.
    - (b) Obtain two equivalent conditions for a topological space to be compact, in terms of its filters.
21. (a) Prove that the filter associated with a universal set is an ultrafilter is a universal set. Prove that both converses also hold.
    (b) Prove : A topological space is Hansdorff iff limits of all nets in it are unique.
22. Obtain four characterisations of countable compactness in T_1—spaces.

$(3 \times 5 = 15)$