QP CODE: 23144636

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023

Third Semester

Faculty of Science

CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

59928569

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Exponential Fourier transform, Fourier sine transform, Fourier cosine transform, Laplace transform and Mellin transform.
2. Define convolution of f and g. Also show by an example that Lebesgue integrability of f and g alone will not give a convolution integral of f and g.
3. Show that if $\mathbf{F}(t) = \mathbf{f}(\mathbf{c} + t\mathbf{u})$, then $\mathbf{F}'(t) = \mathbf{f}'(\mathbf{c} + t\mathbf{u}; \mathbf{u})$, if either derivative exists.
4. Show that the existence of total derivative of a function $\mathbf{f} : S \to \mathbf{R}^m, S \subseteq \mathbf{R}^n$ at $\mathbf{c} \in S$, implies the existence of directional derivative $\mathbf{f}'(\mathbf{c}; \mathbf{u})\ \forall\ \mathbf{u} \in \mathbf{R}^n$.
5. Let $\mathbf{f} : S \to \mathbf{R}^m$ be differentiable at each point of an open connected subset S of $\mathbf{R}^n$. Show that if $\mathbf{f}'(\mathbf{c}) = 0; \forall c \in S$, then $\mathbf{f}$ is constant on S.
6. Let $f(z) = e^z$. Verify that $J_f(z) \neq 0$ for all $z$in C but f is not one- one.
7. State inverse function theorem.
8. Define a Stationary point and a Saddle point.
9. Define flip of a linear operator. Give an example.
10. $Define\ differential\ form\ of\ order\ k.\ Write\ standard\ presentation\ of$ $\omega = x_1 dx_2 \wedge dx_1 \wedge dx_3 - x_2 dx_3 \wedge dx_2 \wedge dx_1.$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let f be a real valued and continuous function on $[a, b]$. Then for every $\epsilon > 0$, there is a polynomial p, such that $|f(x) - p(x)| < \epsilon$ for every x in $[a, b]$.

12. Find the Fourier series for $f(x) = \frac{x^2}{8}, 0 \leq x \leq 2\pi$.

13. a. Define matrix of linear function.
    b. Show that if S, T are linear functions with domain of S containing range of T the matrix of composite function SoT, is the product of matrices of linear functions S, T.

14. Compute the gradient vector $\nabla f(x, y)$ at those points $(x, y) \in \mathbf{R}^2$ if
    a. $f(x, y) = x^2y^2log(x^2 + y^2)\ if\ (x, y) \neq (0, 0), f(0, 0) = 0.$
    b. $f(x, y) = xysin(x^2 + y^2)$

15. Define the Jacobian determinant of a function f on R^n.
    If $f = u + iv$ is a complex valued function with a derivative at a point z in C, Prove that $J_f(z) = |f'(z)|^2$

16. (a) Define Quadratic form. When will you say that a quadratic form is positive definite.
    (b) Find the saddle point of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

17. $For\ every\ f \in C(I^k), show\ that\ L(f) = L^{'}(f).$

18. $(a) If\ \gamma(t) = (acost, bsint),\ 0 \leq t \leq 2\pi, then\ find\ \int_\gamma xdy\ and\ \int_\gamma ydx.$
    $(b) If\ \Phi(r, \theta, \phi) = (x, y, z)\ where\ x = rsin\theta cos\phi,\ y = rsin\theta sin\phi,\ z = rcos\theta\ and\ D\ is\ the 3 - cell$
    $defined\ by\ 0 \leq r \leq 1,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi, then\ show\ that\ \int_\Phi dx \wedge dy \wedge dz = \frac{4\pi}{3}.$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove that the Fourier transform of a concolution $f * g$ is the product of the convolutions of f and of g.

20. State and prove the matrix form of the chain rule.

21. (a) If both partial derivatives $D_r f$ and $D_k f$ exist in an n-ball $B(c)$ and if both $D_{r,k} f$ and $D_{k,r} f$ are continuous at c. Prove that $D_{r,k} f(c) = D_{k,r} f(c)$.
    (b) If both partial derivatives $D_r f$ and $D_k f$ exist in an n- ball $B(c; \delta)$ and if both are differentiable at c. Prove that $D_{r,k} f(c) = D_{k,r} f(c)$.

22. *Suppose* K *is a compact subset of* R^n *and* $\{V_\alpha\}$ *is an open cover of* K.
Show that there exist functions $\psi_1, \psi_2, \ldots\ldots, \psi_s \in C(R^n)$ *such that*
a) $0 \leq \psi_i \leq 1$ *for* $1 \leq i \leq s$
b) Each ψ_i *has its support in some* V_α.
c) $\psi_1(x) + \psi_2(x) + \ldots\ldots + \psi_s(x) = 1, \ \forall x \in K.$

(2×5=10 weightage)