G 2776 (Pages : 2) Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2014

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. If a space X has the property that for any two mutually disjoint closed subsets A and B of it, there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ at all points of B, then show that X is normal.

2. Define the terms box and wall in a set $X = \underset{i \in I}{\pi} X_i$, where $\{X_i : i \in I\}$ is an indexed family of sets. Prove that if a product is non-empty, then each projection function is onto.

3. Prove that in general the box topology is stronger than the product topology and that the two coincide when the index set is finite.

4. Define the evaluation function of the indexed family of functions. Prove that the evaluation function of a family of functions is one-one if and only if the family distinguishes points.

5. Define cluster point of a net. Let $S : D \to X$ be a net and F a cofinal subset of S. If $S/F : F \to X$ converges to a point x in X, then show that x is a cluster point of S.

6. Define a filter on a set X. Also define the term filter base prove that intersection of any family of filters on a set is again a filter on that set.

7. Define a locally compact space. Prove that every locally compact, Hausdorff space is regular.

8. Prove that every continuous, real valued function on a countably compact space is bounded and attains its extrema.

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Let A be a closed subset of a normal space X and $f : A \to [-1, 1]$ is a continuous function. Show that there exists a continuous function $F : X \to [-1, 1]$ such that $F(x) = f(x)$ for all $x \in A$.
10. Prove that if the product is non-empty, then each co-ordinate space is embeddable in it.
11. Prove that a topological space is completely regular if and only if the family of all continuous real valued functions on it distinguishes points from closed sets.
12. Show that a topological space is Hausdorff if and only is limits of all nets in it are unique.
13. Let X, Y be topological spaces, $x_0 \in X$ and $f : X \to Y$ a function. Show that f is continuous at *if* x_0 and only if whenever a net S converges to x_0 in X, the net $f \circ S$ converges to $f(x_0)$ in Y.
14. Prove that every filter is contained in an ultrafilter.
15. Prove that every countably compact metric space is second countable.
16. Prove that the one-point compactification of a space is Hausdorff if and only if the space is locally compact and Hausdorff.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Prove that a product space is connected if and only if each co-ordinate space is connected.
    (b) Prove that a product of topological space is completely regular if and only if each co-ordinate space is completely regular.
18. State and prove Urysohn's lemma.
19. State and prove Urysohn's metrisation theorem.
20. For a topological space, prove that the following are equivalent :
    (a) X is compact.
    (b) Every net in X has a cluster point in X.
    (c) Every net in X has a convergent subnet in X.
21. (a) Prove that a topological space is compact if and only if every ultrafilter in it is convergent.
    (b) Let $\{X_i, i \in I\}$ be a collection of non-empty spaces and let X be its topological product. Prove that X is compact if and only if each X_i is compact.
22. Prove that a subspace of a locally compact Hausdorff space is locally compact if and only if it is open in its closure.

$(3 \times 5 = 15)$