QP CODE: 23002637

23002637

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2023

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

ECC8C5DC

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. State Hadamard's three circle theorem.
2. If $|a| < R$, then evaluate $\int\limits_{|z|=R} \frac{(R^2-|a|^2)}{|z-a|^2} d\theta$.
3. Find the coefficient of z^7 in the expansion of $tanz$ as a Taylor's series.
4. Prove that $\Gamma(n) = (n-1)!$.
5. Define order of an entire function. Give an example of an entire function of order 1.
6. *Define Riemann zeta function and give a connection between $\zeta(s)$ and collection of prime numbers.*
7. *Prove that when zeta function is extended to the whole plane its only pole is a simple pole at s=1 with residue 1.*
8. Let $\mathcal{F}$ be a normal family of functions in Ω with values in a metric space S. Prove that $\mathcal{F}$ is equicontinuous on every compact subset $E \subseteq \Omega$.
9. State and prove the Legendre's relation for the ζ - function.
10. Prove that $\frac{\wp'(z)}{\wp(z)-\wp(u)} = \zeta(z-u) + \zeta(z+u) - 2\zeta(z)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that if $f(z)$ is analytic in a region Ω, then $\overline{f(\overline{z})}$ is analytic in $\Omega^* = \{\overline{z}; z \in \Omega\}$.

12. *Prove that if V is subharmonic in a region Ω then the function V' defined as P_v in Δ and V outside of Δ is also subharmonic, where Δ is an open disk whose closure is contained in Ω .*

13. State Mittag-Leffler's theorem. Prove that $\pi cot\pi z = \frac{1}{z} + \sum_{n\neq 0}(\frac{1}{z-n} + \frac{1}{n})$.

14. Prove that a necessary and sufficient condition for the absolute convergence of the product $\Pi_1^{\infty}(1 + a_n)$ is the convergence of the series $\Sigma_1^{\infty}|a_n|$.

15. *Find the sum of residues of the function* $f(z) = \frac{(-z)^{s-1}}{e^z-1}$.

16. *Define a normal family. Prove that a sequence of functions in $\mathcal{F}$ converges uniformly to f on compact subsets of Ω if it converges to f with respect to the distace function ρ in $\mathcal{F}$.*

17. Let Ω be a simply connected region other than the complex plane. Prove that the Riemann mapping from Ω to the unit disk is onto.

18. State and prove the boundary behavior theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State *Harnack's inequality* and prove *Harnack's Principle.*

20. If $f(z)$ is analytic in the annulus $R_1 < |z-a| < R_2$ and z is any point in the annulus, then prove that $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n + \sum_{n=1}^{\infty} b_n(z-a)^{-n}$ where $a_n = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^{n+1}}$ and $b_n = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^{-n+1}}$, C is the circle $|z-a| = R, R_1 < R < R_2$.

21. (i) Prove that the Zeta function has no zeros in the half plane $\sigma > 1$.
(ii) Describe the various types of zeros of the Zeta function.

22. (a) Prove that a discrete module consists either of zero alone, of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$, or of all linear combinations $n_1\omega_1 + n_2\omega_2$, with non real ratio $\frac{\omega_2}{\omega_1}$ where n_1, n_2 are integers.
(b) Prove that any two bases of the period module are connected by a unimodular transformation.

(2×5=10 weightage)