G 2793 (Pages : 2) Reg. No.................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C09—PARTIAL DIFFERENTIAL EQUATIONS

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Define : Pfaffian differential Equation. When is it exact ?

2. Find the integral curves $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

3. Define compatible systems of first order partial differential equations.

4. Find the complete integral of $g(x, p) = h(y, q)$.

5. What are characteristic curves ? Give an example.

6. Explain how second order equations are classified with example.

7. Show that $u_{xx} + 2u_{yz} + \cos x \, u_z - e^{y^2} u = \cosh z$ is hyperbolic.

8. Write down the elementary solution of the Laplace equation $\nabla^2 \psi = 0$.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Show that $\left(x^2 z - y^2\right) dx + 3xy^2 \, dy + x^3 \, dz = 0$ is integrable and hence solve.

10. Find the integral surface passing through the circle $z = 1$, $x^2 + y^2 = 1$ of the differential equation $(x - y)p + (y - x - z)q = z$.

Turn over

11. Find the complete integrals of $f(p,q)=0$ and $f(z,p,q)=0$.

12. Find the integral surface of $p^2x+qy-z=0$ containing the initial line $y=1, x+z=0$.

13. Reduce $(n-1)^2\ u_{xx}-y^{2n}\ u_{yy}=ny^{2n-1}\ u_y$ to canonical form where $n>1$.

14. Solve $r+s-2t=e^{x+y}$.

15. Show that $x^2+y^2+z^2=cx^{2/3}$ can form an equipotential family of surfaces.

16. Describe Monge's method. Use it to solve $r=t$.

$(5 \times 2 = 10)$

Part C

Answer any **five** *questions.*
Each question carries weight 5.

17. Find the complete integral of $p^2x+qy-z=0$ and derive the equation of the integral surface containing the line $y=1, x+z=0$.

18. State Pfaffian differential equation in its general form and obtain necessary and sufficient condition for it to be integrable.

19. Describe Charpit's method. Use it to find the complete integral of $f=\left(p^2+q^2\right)y-qz=0$.

20. By Jacobi's method solve :

$$z^2+zu_x-u_x^2-u_y^2=0.$$

21. Reduce :

$$x\,u_{xx}+2\sqrt{xy}\,u_{xy}+y\,u_{yy}-u_x=0$$

to Canonical form and solve if possible.

22. Obtain the solution valid when $x, y>0, xy>1, \dfrac{\partial^2 z}{\partial x \partial y}=\dfrac{1}{x+y}$ such that $z=0, p=\dfrac{2y}{x+y}$ on the hyperbola $xy=1$.

$(3 \times 5 = 15)$