G 2779 (Pages : 3) Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2014

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C10—REAL ANALYSIS

(2012 admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Show by an example that boundedness of f' is not necessary for f to be of bounded variation.
2. Define a rectifiable path.
3. If $f_1 \in \mathscr{R}(\alpha)$ and $f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$. Show that $f_1 + f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$.
4. Show that if $f \in \mathscr{R}(\alpha)$ on $[a, b]$, then $|f| \in \mathscr{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right| < \int_a^b |f| \, d\alpha$.
5. For $n = 1, 2, 3, \ldots, x$ real, put $f_n(x) = \frac{x}{1+nx^2}$. Show that $\{f_n\}$ converges uniformly to a function f and that the equation $f'(x) = \lim_{n\to\infty} fn'(x)$ is correct if $x \neq 0$, but false if $x = 0$.
6. For $m, n = 1, 2, 3 \ldots$ let $S_{m,n} = \frac{m}{m+n}$. Show that $\lim_{m\to\infty} \lim_{n\to\infty} S_{m,n} \neq \lim_{n\to\infty} \lim_{m\to\infty} S_{m,n}$.
7. Show that $\lim_{x\to\infty} x^n e^{-x} = 0$ for every n.
8. Show that $\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{inx} dx = \begin{cases} 1 \text{ if } n = 0 \\ 0 \text{ if } n = \pm 1, \pm 2 \ldots \end{cases}$

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Show that if f is monotonic on $[a, b]$, then the set of discontinuities of f is countable.

10. If $c \in (a,b)$, then show that $\wedge_f(a,b) = \wedge_f(a,c) + \wedge_f(c,b)$.

11. State and prove fundamental theorem of calculus.

12. If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

13. If X is a metric space, and $\mathbb{C}(X)$ denote the set of all complex valued continuous bounded functions on X, then show that $\mathbb{C}(X)$ is a complete metric space under the metric induced by the supremum norm.

14. If $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$ and if $f(x) = \sum_{n=1}^{\infty} f_n(x), a \le x \le b$, the series converging uniformly on $[a, b]$, then show that $\int_a^b f d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n d\alpha$.

15. Suppose the series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| < R$, and define $f(x) = \sum_{n=0}^{\infty} C_n x^n, |x| < R$. Prove that $\sum_{n=0}^{\infty} C_n x^n$ converges uniformly on $[-R + \in, R - \in]$ for any $c > 0$ and f is continuous and differentiable in $(-R, R)$ and $f'(x) = \sum_{n=1}^{\infty} nC_n x^{n-1}, |x| < R$.

16. Suppose $a_0, a_1, \ldots, a_n$ are complex numbers, $n \ge 1, a_n \ne 0, P(z) = \sum_0^n a_k z^k$. Show that P $(z) = 0$ for some complex number z.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Let f be of bounded variation on $[a, b]$. If $x \in (a, b)$ and $V(x) = V_f(a, x)$ and $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Also show that the converse is true.

18. (a) Suppose $f \in \mathscr{R}(\alpha)$ on $[a, b]$ $m \leq f \leq M, \phi$ is continuous $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Show that $h \in \mathscr{R}(\alpha)$ on $[a, b]$.

(b) Show that $\underline{\int_a^b} f \, d\alpha \leq \overline{\int_a^b} f \, d\alpha$.

19. Suppose α increases monotimically and $\alpha' \in \mathscr{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Prove that $f \in \mathscr{R}(\alpha)$ if and only if $f \alpha' \mathscr{R}$. Also show that $\int_a^b f \, d\alpha \leq \int_a^b f(x) \alpha'(x) dx$.

20. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n\}$ converges uniformly on $[a, b]$, then show that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and $f'(x) = \lim_{n \to \infty} fn'(x), a \leq x \leq b$.

21. Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{f \to x} f_n(t) = A_n$, $n = 1, 2, 3, \ldots$ Show that $\{A_n\}$ converges and $\lim_{f \to x} f(t) = \lim_{n \to \infty} A_n$.

22. State and prove Parseval's theorem.

$(3 \times 5 = 15)$