G 2790 (Pages : 3) Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C06—ABSTRACT ALGEBRA

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Find the order of $(8, 4, 10)$ in $z_{12} \times z_{60} \times z_{24}$.

2. Give an example to show that a polynomial irreducible over a field need not be irreducible over a larger field.

3. Find the basis of $Q\left(\sqrt[3]{2}, \sqrt{3}\right)$ over Q.

4. Prove or disprove : If α and β are constructible real numbers then so is α/β.

5. For a prime number p, every group G of order p^2 is abelian—Prove.

6. Explain : (i) Class equation ; (ii) Conjugate class.

7. Find the order of $G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$.

8. Prove or disprove : $Q\left[\sqrt{2}, \sqrt{3}\right]$ is separable over Q.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that a direct product of abelian groups is abelian.

10. Define the ring of polynomials $R[x]$ and verify the axioms of ring. Show also that $R[x]$ is commutative if R is commutative.

Turn over

11. Show that all finite fields must have prime power order.

12. Prove in detail $Q\left(\sqrt{3}+\sqrt{7}\right)=Q\left(\sqrt{3}, \sqrt{7}\right)$.

13. If H and k are finite subgroups of a group G, prove :

$$|Hk| = \frac{(|H|)\,|k|}{|H \cap k|}.$$

14. Show that there are no simple groups of order $255 = (3)\,(5)\,(17)$.

15. Show that $f(x) \in F[x]$ has no zero of multiplicity >1 if and only if $f(x)$ and $f'(x)$ have no common factor in $\overline{F}[x]$ of degree > 0.

16. Show that an automorphism of a splitting field E over F of a polynomial $f(x) \in F[x]$ permutes the zeros of $f(x)$ in E.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Establish Eisenstein criterion .

    (b) Establish niqueness of factorization in $F[x]$.

18. (a) Show that $x^3 + 17x + 36$ is irreducible over $Q[x]$.

    (b) Find the number of irreducible polynomials in $Z_p[x]$ where p is prime.

19. (a) Show that the set of all constructible red numbers form a subfield of real numbers.

    (b) Show algebraically that it is possible to construct an angle of 30°.

20. Let F be a field and let α and β be algebraic over F with deg $(\alpha, F) = n$. Prove that the map $\psi_{\alpha,\beta} : F(\alpha) \to F(\beta)$ defined by

$$\psi_{\alpha,\beta}\left(c_0 + c_1\alpha + \ldots\ldots + c_{n-1}\,\alpha^{n-1}\right) = c_0 + c_1\beta + \ldots\ldots + c_{n-1}\beta^{n-1}$$

for $c_i \in F$ is an isomorphism of $F(\alpha)$ onto $F(\beta)$ iff α and β are conjugate over F.

21. State and prove Frobenius Automorphism Theorem.

22. State and prove the main theorem of Galois theory.

(3 × 5 = 15)