G 2351 (Pages : 2) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2015

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT04C16—SPECTRAL THEORY

(Programme—Core—Common for all)

[2012 Admission onwards—Regular/Supplementary]

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define closed operator with an example.
2. Show that strong convergence always implies weak convergence of operators.
3. If $\|x - e\| < 1$, show that x is invertible. Also obtain an expression for x^{-1}.
4. Give an example of a normed space which is not Banach.
5. Establish the uniqueness of the adjoint of an operator.
6. Give an example of a compact linear operator on normed space.
7. Give an example of self adjoint operator and find its spectrum.
8. Define projection operator with two examples.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each questions has weight 2.

9. Let $T_n \in B(X, Y)$ where X is a Banach space. If (T_n) is strongly operator convergent show that $(\|T_n\|)$ is bounded.
10. Define strong operator convergence and uniform operator convergence. Further establish the relationship between them with illustrations.

Turn over

11. Show that closedness does not imply boundedness of a linear operator. Establish the converse also.
12. Establish the linear independence property of eigen vectors.
13. Show that the set of all linear operators on a vector space into itself forms an algebra.
14. Show that the set of all complex matrics of the form $x = \begin{bmatrix} \alpha & \beta \\ 0 & 0 \end{bmatrix}$ forms a sub-algebra of the algebra of all complex 2×2 matrics and find $\sigma(x)$.
15. Show that a compact linear operator on a normed space X can be extended to the completion of X, the extended operator being linear and compact.
16. (a) Give an example of a self adjoint positive operator. Justify.
    (b) Define the residual spectrum of a bounded linear operator T and find it if T is defined on a complex Hilbert space.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove Banach fixed point theorem ? Also explain graph of an operator with examples. Is the completeness condition in the fixed point theorem necessary ? Explain.
18. Obtain a formula for the spectral radius of an operator on a complex Banach space.
19. Let $T : X \to X$ be a compact linear operator on a Banach Space X. Prove that every spectral value $\lambda \neq 0$ of T is an eigen value of T.
20. (a) State and prove the conditions under which the limit of a sequence of compact operators is compact.
    (b) State and prove the conditions for which the inverse of adjoint equals the adjoint of inverse.
21. (a) Let $T : l^2 \to l^2$ be given by $T(x_1, x_2, ...) = (0, 0, x_3, x_4, ...)$. Find the square root of T.
    (b) Give an example of a bounded self adjoint operator on a Hilbert space and find its spectrum.
22. State and prove five properties of projections.

$(3 \times 5 = 15)$