QP CODE: 22001451

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2022

First Semester

CORE - ME010104 - REAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

AAE6E8D1

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that the set of discontinuities of a monotonic function is countable.
2. Let f be of bounded variation on $[a, b]$ and assume that f is bounded away from zero; that is, there exists a number m such that $0 < m \leq | f(x)|$ for all $x \in [a, b]$.
   Prove that $g = \dfrac{1}{f}$ is of bounded variation on $[a, b]$, and $V_g \leq \dfrac{V_f}{m^2}$.
3. *Define upper and lower Riemann Stieltjes integral.*
4. *If U (P, f, α) - L (P, f, α) < ε for a partition P = { x_0, x_1,x_2,...,x_n} and if s_i , t_i are arbitrary points in [x_{i-1} , x_i] then prove that* $\sum_{i=1}^{n} |f(s_i) - f(t_i)|\Delta\alpha_i < \varepsilon$.
5. *Define integration of vector valued functions.*
6. When is a sequence of functions said to be pointwise convergent ?
7. If the sequence of functions $\{f_n\}$ and $\{g_n\}$converge uniformly on a set E, prove that $\{f_n + g_n\}$ converges uniformly on E.
8. State Cauchy criterion for uniform convergence of sequences of functions.
9. If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3, \ldots,$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K.
10. Define the exponential function $E(z)$ and prove that $E(0) = 1$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let f be continuous on $[a, b]$ then prove that f is of bounded variation on $[a, b]$ if and only if f can be expressed as the difference of two increasing functions.

12. Let $f:[a, b] \rightarrow \mathbb{R}^n$. Then prove that f is rectifiable if and only if each of the components f_k of f is of bounded variation on $[a, b]$.
Also prove that if f is rectifiable then
$V_k(a, b) \leq \Lambda_f(a, b) \leq V_1(a, b)+\cdots+V_n(a, b), \qquad k=1,2, \ldots, n,$
where $V_k(a, b)$ denotes the total variation of f_k on $[a, b]$.

13. *If f is continuous on [a, b] then show that $f \in \mathscr{R}(\alpha)$.*

14. If a < s < b , f is bounded on [a,b] , f is continuous at ***s*** and α(x) = *I*(x-***s***), where *I* is the unit step function, then prove that $\int_a^b f d\alpha=f(s)$.

15. Give an example to show that a sequence of continuous functions may converge to a continuous function, although the convergence is not uniform.

16. Obtain a series from $\phi(x)=|x|,(-1 \leq x \leq 1)$ and $\phi(x+2)=\phi(x)$ for all real x , which converges uniformly on R^1.

17. Suppose $\{f_n\}$ is an equicontinuous seqeuce of functions on a compact set K, and $\{f_n\}$ converges pointwise on K. Prove that $\{f_n\}$ converges uniformly on K.

18. State and prove Taylor's theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Define $s(x)=\Lambda_f(a, x)$ for $x \in[a, b]$ and let $s(a)=0$ for a rectifiable path f defined on $[a, b]$. Then prove that the function f is increasing and continuous on $[a, b]$ and if there is no subinterval of $[a, b]$ on which f is constant, then s is strictly increasing on $[a, b]$.
(ii) Let $f:[a, b] \rightarrow R^n$ and $g:[c, d] \rightarrow R^n$ be two paths in R^n, each of which is one to one on its domain. Then prove that $\mathbf{f}$ and $\mathbf{g}$ are equivalent if and only if they have the same graph.

20. *(i) If $f_1 \in \mathscr{R}(\alpha)$ and $f_2 \in \mathscr{R}(\alpha)$ on [a,b] then prove that $f_1+f_2 \in \mathscr{R}(\alpha)$ and* $\int_a^b(f_1+f_2) d\alpha=\int_a^b f_1 d\alpha+\int_a^b f_2 d\alpha$.
(ii) If $f \in \mathscr{R}(\alpha)$ on [a,b] and if a < c < b then prove that $f \in \mathscr{R}(\alpha)$ on [a, c] and [c, b] and
$\int_a^c f d\alpha+\int_c^b f d\alpha=\int_a^b f d\alpha$

21. State and prove the condition for the uniform convergence of $f_n^{\prime} \rightarrow f^{\prime}$ if $f_n \rightarrow f$ uniformly.

22. State and prove Weierstrass approximation theorem.

(2×5=10 weightage)