QP CODE: 22001760

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, AUGUST 2022

Fourth Semester

Elective - ME800403 - COMBINATORICS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

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Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Stirling's numbers of the first kind and find the value of $s(r, r-1)$.
2. Using bijection principle Show that $\binom{n}{r} = \binom{n}{n-r}$ for $n, r \in N with\; r \leq n$
3. In how many ways can distribute r identical objects into n distinct boxes , if each box can hold any number of objects
4. Define a clique and K-clique with example
5. Find the upper bound for R(3,4)
6. What is E(m) and $\omega(m)$ in GPIE in usual notation
7. Define Strinlings number of second kind S(n,m).Express it as a sum of a finite series
8. Prove that $\lim_{n\to\infty} \frac{D_n}{n!} = e^{-1}$
9. Define Ferrers diagram with an example? When will you say that two partitions of a positive integer are conjugate to each other
10. What is mean by solution of a recurrence relation? Give example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. A) Show that for any$n \in N$, the number of positive divisors of n^2 is always odd
B)Show that the number of positive divisors of " $111...111$" is even, if there are 1992 ,$1's$ in the number

12. Find the number of ways to pave 1×7 rectangle by 1×1,1×2 and 1×3 blocks, assuming that blocks of same size are indistinguishable

13. Let A be a set of m positive integers where $m \geq 1$. Show that there exist a non empty subset B of A such that the sum $\sum(x|x \in B)$ is divisible by m

14. Show that R(2,q)=q and using suitable reccursive formula evaluate an upper bound for R(3,3)

15. A) State simplest form of Principle of inclusion and exclusion for two finite sets and extend it for three finite sets

B)A group of 102 students took examinations in Chinese, English and Mathematics. Among them 92 passed Chinese, 75 English and 63 Mathematics; atmost 65 passed Chinese and English, atmost 54 Chinese and Mathematics and atmost 48 English and Mathematics. Find the largest possible number of the the students that could have passed all the three subjects

16. Applying GPIE ,find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 40$ where $x_1 \leq 9, x_2 \leq 15, x_3 \leq 15$

17. Express the generating function for the sequence (c_r) where $c_r = \sum_{i=1}^{r} i^2$ for each $r \in N^*$ in a form not involving any series.And hence show that $\sum_{i=1}^{r} i^2 = \binom{r+1}{3} + \binom{r+2}{3}$

18. A man wishes to climb an n-step staircase.Let a_n denote the number of ways that this can be done if in each step he can cover either one step or two steps.Find a reccurence relation for (a_n) and hence solve it by finding initial conditions

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. A) Find the number of ways of arranging the 26 letters in the English alphabets in a raw such that there are exactly 5 letters between x and y

B)Find the number of odd integers between 3000 and 8000 in which no digit is repeated

C)In how many ways can boys and 3 girls be seated around a table so that no girls are adjacent

20. A) Show that among any points in an equilateral trainagle of unit side length,there are 2 whose distance is almost half units apart

B) Given any set C of n+1 distinct points on the circumferrence of a unit circle show that there exist two distinct points a and b such that the distance between them doesnot exceed $2sin\frac{\pi}{n}$

21. State and prove Generalised Principle of Inclusion and Exclusion

22. (a) In how many ways can 4 of the letters from PAPAYA be arranged?

(b) For each $r \in N^*$, let a_r denote the number of r-digit quaternary sequences in which each of the digits 2 and 3 appears atleast once. Find a_r.

(2×5=10 weightage)