QP CODE: 22000699

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2022

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

65FA94DE

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Poisson integral.
2. State Reflection principle.
3. Write the Taylor series expansion of arc tanz about the origin.
4. Prove that a non zero entire function can be represented in the form $f(z) = e^{g(z)}$ where $g(z)$ is an entire function.
5. Define the genus of an entire function. Give an example of an entire function with genus 1.
6. *Define Riemann zeta function. Prove that number of primes is infinite.*
7. Define a normal family of functions in a region Ω.
8. State Arzela's theorem.
9. Write the Riemann mapping theorem.
10. Prove that $\zeta(z)$ is semi periodic with periods ω_1 and ω_2 .

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *If $U(z)$ is harmonic in $|z| < \rho$ and continuous in $|z| \leq \rho$ then prove that $\frac{\rho-r}{\rho+r}U(0) \leq U(z) \leq \frac{\rho+r}{|\rho-r}U(0)$ for every z with $|z| = r < \rho$.*

12. Prove that *a continuous function* $V(z)$ *is subharmonic in* Ω *iff it satisfies the inequality* $V(z_0) \leq \frac{1}{2\pi}\int_0^{2\pi} V(z_0 + re^{i\theta})d\theta$ *for every disk* $|z - z_0| \leq r$ *, contained in* Ω *.*

13. Write the general form of a Laurent series for $f(z)$ which is analytic in the annulus $R_1 < |z-a| < R_2$. Derive the Laurent series of $f(z) = \frac{1}{(1+z)}$ in $1 < |z| < \infty$.

14. State Mittag-Leffler's theorem.Using it evaluate the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

15. *Prove that* $\xi(s) = \xi(1-s)$ *where* $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$.

16. *Write a short note on zeros of the Riemann zeta function.*

17. Suppose that the boundary of a simply connected region Ω contains a line segment γ as a one sided free boundary arc. Then prove that the function $f(z)$ which maps Ω onto the unit disk can be extended to a function which is analytic and one- to- one on $\Omega \cup \gamma$. Prove that the image of γ is an arc γ' on the unit circle.

18. State and prove the addition formula for the $\wp$ - function.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) State and prove the mean value property of harmonic functions.

    (b) If $u(z)$ *is harmonic in a disc* $|z - z_0| < \rho$, then prove that $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + \rho e^{i\theta})d\theta$.

20. Derive a recurrence formula for Gamma function. Find the residue of $\Gamma(z)$ at the poles $z = -n$.

21. (i) Define the Riemann Zeta function.
    (ii) Derive the functional equation of the Zeta function.
    (iii) Prove that $\xi(s) = \frac{1}{2}s(1-s)\pi^{\frac{-s}{2}}\Gamma(\frac{s}{2})\zeta(s)$ is an entire function.

22. (a) Prove that an elliptic function without poles is a constant.
    (b) Prove that the sum of residues of an elliptic function is zero.
    (c) Prove that a non constant elliptic function has equally many zeros and poles.

(2×5=10 weightage)