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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2018

Second Semester

Faculty of Science

Branch I (A) : Mathematics

MT02C10—REAL ANALYSIS

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Explain whether the graph given by :

   $f(x) = x\cos(\pi/2x)$, for $x \neq 0$ and $f(0) = 0$ is rectifiable.

2. Establish the condition for the paths $\vec{f}$ and $\vec{g}$ to be equivalent.

3. Use the definition of R–S integral to evaluate $\int_a^b d\alpha(x)$.

4. Define upper and lower Stieltjes sums.

5. Define equicontinuous family of functions.

6. Construct sequences $\{f_n\}$, $\{g_n\}$ which converge uniformly on some set E, but $\{f_n g_n\}$ does not converge uniformly on E.

7. Evaluate $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$.

8. Prove that every continuous mapping f of $\bar{D}$ into $\bar{D}$ has a fixed point in $\bar{D}$.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Give example to show that a continuous function need not be a function of bounded variation. Give proof.

10. Show that a function of bounded variation is bounded.

11. Suppose f is a bounded real function on $[a, b]$ and $f^2 \in \mathrm{R}$ on $[a, b]$. Does it follow that $f \in \mathrm{R}$?

    Does the answer change if we assume that $f^3 \in \mathrm{R}$?

12. Suppose α increases monotonically on $[a, b]$, g is continuous and $g(x) = \mathrm{G}^1(x)$ for $a \leq x \leq b$. Prove that :

    $$\int_a^b \alpha(x)\, g(x)\, dx = \mathrm{G}(b)\,\alpha(b) - \mathrm{G}(a)\,\alpha(a) - \int_a^b \mathrm{G}\, d\alpha.$$

13. Give examples :

    (a) Uniformly convergent sequence of functions.

    (b) Absolutely convergent sequence of functions.

    Prove your answers.

14. If $\{f_n\}$ is a sequence of continuous functions on E and $f_n \to f$ uniformly on E, Prove that f is continuous on E.

15. Evaluate $\lim\limits_{x \to 0} \dfrac{e - (1+x)^{1/x}}{x}$.

16. Find the Fourier coefficients :

    $f(x) = x$ if $0 \leq x < 2\pi$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Show that a continuous function on $[a, b]$ is of bounded variation if and only if f can be expressed as the difference of two increasing continuous functions.

18. Establish necessary and sufficient condition for $f \in \mathrm{R}(\alpha)$ on $[a, b]$.

19. State and prove the theorem on the differentiation under integral sign.

20. Let $\{f_n\}$ be a sequence of continuous functions which converges uniformly to a function f on a set E. Prove that :

$$\lim_{n \to \infty} f_n(x_n) = f(x)$$

for every sequence of points $x_n \in \mathrm{E}$ such that $x_n \to x$ and $x \in \mathrm{E}$. Prove that converse is not true.

21. State and prove Taylor's theorem for a continuous function defined on R. Also use Maclaurin's series to evaluate $\lim_{x \to 0} \frac{x - \sin x}{\tan x - x}$.

22. Establish the existence of a nowhere differentiable function.

(3 × 5 = 15)