



QP CODE: 23145322



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Reg No :

Name :

M Sc DEGREE (CSS) EXAMINATION, DECEMBER 2023

First Semester

CORE - ME010103 - BASIC TOPOLOGY

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

D98FE0CB

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. What you mean by metrisable topology, Give an example with justification?
2. Define a base of a topological space? Justify with example
3. Define subspace of a topological space .How will you write an open set in a space using its subbase elements?Why?
4. *Let A be a subset of a Topological Space (X, \mathcal{T}) . Then Show that \bar{A} is a closed set and is the smallest closed subset of X containing A .*
5. Define strong topology.
6. *Define quotient map and quotient topology.*
7. Define a compact subset of a topological space X and give an example
8. Define connected space. If X is a connected space, prove that X cannot be written as the disjoint union of two nonempty closed subsets .
9. Define a path, simple path, closed path in a topological space and a path connected space
10. *Show that every T_1 space is T_0 but not conversely*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.

11. Prove that the collection of all open sets in a metric space form a topology on the same set
12. Show that there exist a unique smallest topology on a non empty set X containing any given family of subsets of X





13. Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.
14. Let $f : X \rightarrow Y$ be a function, where X, Y are topological spaces. Then prove that the following statements are equivalent (i) f is a homeomorphism, (ii) f is a continuous bijection and f is open. (iii) f is a bijection and f^{-1} is open. (iv) There exist a function $g : Y \rightarrow X$ such that f, g are continuous and $g \circ f = id_X$ and $f \circ g = id_Y$
15. Let X, Y be topological spaces, $x \in X$, and $f : X \rightarrow Y$ a function. Suppose X is first countable at x and f is continuous at x and $\{x_n\}$ is a sequence in X . Prove that the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y .
16. Let X_1, X_2 be connected topological spaces. Prove that $X_1 \times X_2$ is connected.
17. Let Y be a Hausdorff space and X be any space. Let f and g be any two maps from X to Y . Show that the set $\{x \in X : f(x) = g(x)\}$ is closed in X
18. Show that regularity is a hereditary property

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. Show that metrisability is a hereditary property
20. Let $f : X \rightarrow Y$ be a function, where \mathcal{I}, \mathcal{U} be topologies on X, Y respectively and $x_0 \in X$. Prove that the following statements are equivalent. (i) f is continuous at x_0 , (ii) the inverse image of every neighborhood of $f(x_0)$ in Y is a neighborhood of x_0 in X , (iii) for every subset $A \subseteq X$, $x_0 \in \bar{A}$ implies $f(x_0) \in \overline{f(A)}$
21. (a) Define Lebesgue number of a cover of a topological space X .
(b) State and prove Lebesgue covering Lemma.
22. Show that locally connectedness is a divisible property

(2×5=10 weightage)

