G 2778 (Pages : 3) Reg. No.................................

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# M.Sc. DEGREE (C.S.S.) EXAMINATION AUGUST 2014

**Second Semester**

Faculty of Science

Branch I (A)—Mathematics

MT 02 C09—PARTIAL DIFFERENTIAL EQUATIONS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

## Part A

*Answer any* **five** *questions.*
*Each question has weight* 1.

1. Determine whether the equation $ydx + xdy + 2zdz = 0$ is integrable.

2. Eliminate the constants $a$ and $b$ from the equation $z = (x + a)(y + b)$.

3. Prove that along every characteristic strip of the equation $F(x, y, z, p, q) = 0$ the function $F(x, y, z, p, q)$ is a constant.

4. Find a complete integral of the equation $p^2 y(1 + x^2) = qx^2$.

5. Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by $z = \frac{1}{x}\phi(y - x) + \phi'(y - x)$, where $\phi$ is an arbitrary function.

6. Solve the equation $r + s - 2t = e^{x+y}$.

7. What is interior Churchill problem.

8. Prove that $r\cos\theta$ satisfies Laplaces equation, where $r, \theta\ \phi$ are spherical polar co-ordinates

$(5 \times 1 = 5)$

## Part B

*Answer any* **five** *questions.*
*Each question has weight* 2.

9. Find the integral curves of $\frac{dx}{xz - y} = \frac{dy}{yz - x} = \frac{dz}{1 - z^2}$.

Turn over

10. Verify that the equation $z(z+y^2)\,dx + z(x+x^2)\,dy - xy(x+y)\,dz = 0$ is integrable and find its primitive.

11. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$.

12. Find a complete integral of the equation $p^2 x + qy = z$ and hence derive the equation of an integral surface of which the line $y = 1, x + z = 0$ is a generator.

13. Find the solution of the equation $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = x - y$.

14. Reduce the equation $(n-1)^2 \dfrac{\partial^2 z}{\partial x^2} - y^{2n} \dfrac{\partial^2 z}{\partial y^2} = n y^{2n-1} \dfrac{\partial z}{\partial y}$ to canonical form, and find its general solution.

15. Show that the surfaces $x^2 + y^2 + z^2 = cx^{3/2}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

16. By separating the variables, show that the one-dimensional wave equation $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{1}{c^2} \dfrac{\partial^2 z}{\partial t^2}$ has solution of the form A $\exp(\pm inx \pm inct)$ where A and $n$ are constants.

$(5 \times 2 = 10)$

## Part C

*Answer any* **three** *questions.*
*Each question has weight* 5.

17 Prove that a necessary and sufficient condition that the Pfaffian differential equation $\mathrm{X} \cdot \mathrm{dr} = 0$ should be integrable is that $\mathrm{X} \cdot \mathrm{curl}\, \mathrm{X} = 0$.

18 If $u$ is a function of $x$, $y$ and $z$ which satisfies the partial differential equation

$(y-z)\dfrac{\partial u}{\partial x} + (z-x)\dfrac{\partial u}{\partial y} + (x-y)\dfrac{\partial u}{\partial z} = 0$. Show that $n$ contains $x$, $y$ and $z$ only in combinations $x + y + z$ and $x^2 + y^2 + z^2$.

19. Explain Charpit's method of solving the partial differential equation $F(x, y, z, p, q) = 0$ and solve the equation $p^2 x + q^2 y = z$ by Charpit's method.

20. Prove that a necessary and sufficient condition that a surface be an integral surface of a partial differential equation is that at each point its tangent element should touch the elementary cone of the equation.

21. Obtain the solution, valid when $x, y > 0$, $xy > 1$, of the differential equation $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x+y}$ such that $z = 0$, $p = 2y/(x+y)$ on the hyperbola $xy = 1$.

22. (a) Solve the equation $pq = x(ps - qr)$.

    (b) If $p > 0$ and $\psi(r) = \int_V \frac{p(r')\,dr'}{|r - r'|}$, where V is the volume which is bounded, prove that $\lim_{r \to \infty} r\,\psi(r) = M$, where $M = \int_V \rho(r')\,dr'$.

(3 × 5 = 15)