F 7734 (Pages : 3) Reg. No.................................

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M.Sc. DEGREE (C.S.S) EXAMINATION, FEBRUARY 2014

First Semester

Faculty of Science

Branch I (A)—Mathematics

MTO IC 02—BASIC TOPOLOGY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Let X be a set and $\{\Im_i, i \in I\}$ be an indexed collection of topologies on X. Show that $\Im = \bigcap_{i \in \Im} \Im_i$ is a topology on X.
2. Show that a subset A of a topological space X is deuse in X if and only if for every non-empty open subset B of X, $A \cap B \neq \phi$.
3. Show that f is continuous if and only if for all $A \subset X$, $f(\bar{A}) \subset \overline{f(A)}$.
4. Show that every second countable space if first countable.
5. Prove that continuous image of a connected space is connected.
6. Show by an example that connectedness need not imply local connectedness.
7. Show that regularity is a hereditary property.
8. Show that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. For a subset A of a topological space X, show that $\bar{A} = A \cup A'$.
10. Prove that a discrete space is second countable if and only if the underlying set is countable.

Turn over

11. Let $f : X \to Y$ be a function, where X and Y are topological spaces. Prove that if f is continuous, then the graph of f is homeomorphic to X.

12. Show that every separable space satisfies the countable chain condition.

13. Show that closure of a connected space is connected. Is the converse true. Justify your answer.

14. Show that every quotient space of a locally connected space is locally connected.

15. Define a T_4 space. Show that all metric spaces are T_4.

16. Define a completely regular space. Give an example. Show that every completely regular space is regular.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Let X be a set, $\mathfrak{I}$ a topology on X and S a family of subsets of X. Show that S is a sub base for $\mathfrak{I}$ if and only if S generates $\mathfrak{I}$.

    (b) If $(X, \mathfrak{I})$ is second countable and $Y \subset X$, then show that any cover of Y by members of $\mathfrak{I}$ has a countable subcover.

18. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

    (b) Prove that continuous image of a compact space is compact.

19. (a) State and prove Lebesgue covering lemma.

    (b) Let X be an uncountable set with the co-countable topology on X. Prove that X is not separable.

20. (a) Prove that every closed and bounded interval is compact.

    (b) Show that union of a collection of connected sub sets of X having a common point is connected.

21. (a) Let X be a completely regular space. Suppose F is a compact subset of X, $\subset$ is a closed subset of X and $F \cap C = \phi$. Prove that there exists a continuous function from X into the unit interval which takes the value 0 at all points of F and the value 1 at all points of F and the value 1 at all points of C.

    (b) Show that every map from a compact space into a T_2 space is closed.

22. (a) For a topological space X prove that the following statements are equivalent :

    (i) X is regular.

    (ii) For any $x \in X$ and any open set G containing x, there exists an open set H containing x such that $\overline{H} \subset G$.

    (iii) The family of all closed neighbourhoods of any point of X forms a local base at that point.

(b) Prove that a subset of R is connected if and only if it is an interval.

$(3 \times 5 = 15)$