QP CODE: 22000701

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2022

Third Semester

Faculty of Science

CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

3FE160A9

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 7x, 0 < x < 2\pi$
2. Define convolution of f and g. Also show by an example that Lebesgue integrability of f and g alone will not give a convolution integral of f and g.
3. Define a linear function. If $\mathbf{f} : \mathbf{R}^n \to \mathbf{R}^m$ is linear then show that $\mathbf{f}'(\mathbf{c}, \mathbf{u}) = \mathbf{f}(\mathbf{u})$ for every $\mathbf{c}$ and every $\mathbf{u}$.
4. State a necessary but not sufficient condition for differentiability of a complex-valued function of a complex variable $f = u + iv$.
5. Let $\mathbf{f} : \mathbf{R}^2 \to \mathbf{R}^3$ be defined by the equation, $\mathbf{f}(x, y) = (sinxcosy, sinxsiny, cosxcosy)$. Determine the Jacobian matrix.
6. Define Jacobian determinant and find the Jacobian determinant for the function $f(z) = z^2 + z$
7. State implicit function theorem
8. Prove that the function $f : R^2 \to R$ defined by $f(x, y) = (y - x^2)(y - 2x^2)$ does not have a local maximum or local minimum at $(0, 0)$
9. $Write\ the\ necessary\ and\ sufficient\ condition\ for\ invertibility\ of\ G'(a),\ a \in E \subset R^n$ $where\ G\ is\ a\ primitive\ mapping.$
10. $Define\ differential\ form\ of\ order\ k.\ Write\ standard\ presentation\ of$ $\omega = x_3 dx_2 \wedge dx_1 \wedge dx_3 - x_2 dx_3 \wedge dx_2 \wedge dx_1 + x_1 dx_1 \wedge dx_3 \wedge dx_2.$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that every real valued and continuous function on a compact interval can be uniformly approximated by a polynomial.

12. If $p > 0, q > 0$, prove that the beta function can be expressed using gamma function as $\mathcal{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$. Evaluate $\mathcal{B}(2,3)$

13. a. Define the total derivative of a function $f : S \to R^m; S \subseteq R^n$ at an interior point $c \in S$. Find the total derivative of a linear function.
b. The existence of the total derivative implies the existence of directional derivatives. True or false. Justify.

14. State and prove mean value theorem.

15. Let $B = B(a,r)$ be an n- ball in R^n and ∂B denote its boundary, $\partial B = \{x : \| x - a \| = r\}$ and let $\overline{B} = B \cup \partial B$ denote its closure . Let $f = (f_1, f_2, \ldots . f_n)$ be continuous on $\overline{B}$ and assume that all the partial derivatives $D_j f_i(x)$ exist if $x \in B$. Assume further that $f(x) \neq f(a)$ if $x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each $x$in B. Prove that $f(B)$ the image of B under f , contains an n- ball with centre at $f(a)$.

16. (a) Define Quadratic form. When will you say that a quadratic form is positive definite.
(b) Find the saddle point of the function $f(x,y) = 7x^2 + \frac{y^3}{3} - 4xy$

17. $Prove\ that\ the\ k\ integrations\ of\ f\ over\ I^k\ is\ independent\ of\ the\ order,\ for\ all\ \ f \in C(I^k).$

18. $Let\ \gamma\ be\ a\ 1-surface\ in\ R^3\ with\ parameter\ domain\ [0,1]\ and\ \omega = xdy + ydx. Then\ prove\ that\ \int_\gamma \omega$ $depends\ only\ on\ the\ endpoint\ of\ the\ curve\ \gamma$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove the theorem which gives the sufficient conditions for representing a function by Fourier integrals (Fourier Integral Theorem).

20. State and prove the chain rule.

21. If both partial derivatives $D_r f$and $D_k f$ exist in an n- ball $B(c,\delta)$ and if both are differentiable at c then Prove that $D_{r,k}f(c) = D_{k,r}f(c)$

22. $State\ and\ prove\ theorem\ on\ partitions\ ofunity.$

(2×5=10 weightage)