G 17003089

17003089

Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, JULY 2017

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C08—ADVANCED COMPLEX ANALYSIS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define absolute convergent series with an example with justification.
2. Find the radius of convergence of $\sum \frac{z^n}{n^n}$.
3. Define entire function with two examples.
4. State Bolzano-Weierstrass theorem. Also explain the notion of relatively compact family.
5. State the theorem on the solution of the Dirichlet problem.
6. Examine whether $\log\left(1+|z|^2\right)$ is sub-harmonic.
7. Give examples for simply periodic and doubly periodic functions with proof.
8. State the formal definition of homotopy.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.
10. Obtain a series expansion for Euler's constant and write its approximate value.
11. Define metric space with an example. If d is a distance function, show that $\delta(a,b) = \frac{d(a,b)}{1-d(a,b)}$ is also a distance function.

Turn over

12. Define genus and order of an entire function and state the relationship between them. Also give an example of entire function.

13. State and prove the theorem on the boundary behaviour of a topological mapping.

14. Characterise a sub-harmonic function by an inequality which generalises the mean-value property of harmonic function.

15. Obtain Legendre's relation.

16. With usual notation prove :

$$p(z) - \frac{1}{z^2} = \sum_{w \neq 0} \left(\frac{1}{(z-w)^2} - \frac{1}{w^2} \right).$$

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Establish Abel's first theorem on uniform convergent, divergent and radius of convergence of derived series.

18. Obtain Jensen's formula. Deduce Poisson-Jensen formula.

19. State and prove The Riemann mapping Theorem.

20. Derive Harnack's inequality and deduce Harnack's principle.

21. Establish the theorem on the existence of the canonical basis.

22. Establish the existence of homotopy group. Also explain why it is sufficient to study the homotopy of closed curves.

(3 × 5 = 15)