

G 2792

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Reg. No.....

Name.....

**M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016**

**Second Semester**

**Faculty of Science**

**Branch I (A) : Mathematics**

**MT 02 C08—ADVANCED COMPLEX ANALYSIS**

**(2012 Admissions)**

**Time : Three Hours**

**Maximum Weight : 30**

**Part A**

*Answer any five questions.*

*Each question has weight 1.*

1. Explain the two representations of a rational function with examples.
2. Prove that a convergent sequence is bounded. Is the converse true ? Why ?
3. Write a note on the zeros of the zeta function.
4. State Arzela-Ascoli theorem and explain.
5. Write down the basic properties of harmonic functions.
6. Write down the theorem on the solution of Dirichlet problem.
7. Differentiate between simply periodic and doubly periodic functions with illustrations.
8. Explain : Direct analytic continuation.

(5 × 1 = 5)

**Part B**

*Answer any five questions.*

*Each question has weight 2.*

9. State and prove Hurwitz theorem.
10. Write down the expansion for  $(1 - z^2)^{-\frac{1}{2}}$  for  $|z| < 1$ . Use it to develop a series for  $\arcsin z$ .

**Turn over**

11. Obtain Jensen's formula.
12. State and prove the theorem on the characterisation of normal family using compactness.
13. Prove that a subharmonic function remains sub-harmonic if the independent variable is subjected to conformal mapping.
14. Show that a continuous function is harmonic if and only it satisfies the mean-value property.
15. Determine all discrete modules. Give proof for your statement.
16. Explain : Germs and sheaves.

(5 × 2 = 10)

### Part C

Answer any **three** questions.

Each question has weight 5.

17. (a) Obtain necessary and sufficient condition for an infinite product to be absolutely convergent.
- (b) Prove that for  $|z| < 1$  :

$$\prod_{n=1}^{\infty} (1 + z^n) = \frac{1}{1 - z}.$$

18. (a) State the double inequality satisfied by genus and the order of an entire function. Explain it.
- (b) Derive Jensen's formula.
- (c) Obtain Poisson-Jensen formula.
19. Obtain Schwarz-Christoffel formula. Also state its another version.
20. Obtain Harnack's principle and Harnack's inequality.
21. (a) Prove that any two bases of the same module are connected by a unimodular transformation.
- (b) State the uniqueness theorem on the analytic continuation and explain.
22. State and prove four properties of elliptic function.

(3 × 5 = 15)