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Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JULY 2017

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Obtain a basis for product topology of Cartesian product of two topological spaces.
2. Explain productive property with an example.
3. State three sets of hypothesis under which normality of a space is assured.
4. Define matrizable space. Give example of a metrizable space and a space not metrizable.
5. Define :
   (a) Filter and ultra filter.
   (b) Net and a subnet.
6. Explain the convergence of filters.
7. Define compact space. Check whether the real line R is compact.
8. Explain locally compact space and Hausdorff space.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Explain a normal space. Show that a closed subspace of a normal space is normal.
10. Show that if the product is non-empty then each co-ordinate space is embeddable in it.
11. Define metric topology with an example and justify your result.
12. Show that X is compact if and only if every net in X has a convergent subnet.
13. Show that the net (x_α) has the point x as an accumulation point if and only if some subnet of (x_α) converges to x.

Turn over

14. Show that every filter is contained in an ultra filter.
15. Explain the formulation of local compactness and describe one more variation of it.
16. Show that every countably compact metric space is second countable.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Establish Urysohn characterisation of normality.
18. Explain :
    (a) Cartesian products of families of sets with an example.
    (b) Tietze characterisation of normality.
19. Establish the Urysohn metrisation theorem.
20. Let $f : X \to Y$. Show that f is continuous if and only if for every convergent net (x_n) in X converging to x, the net $(f(x_n))$ converges to $f(x)$.
21. Let $f : X \to Y$. show that f is continuous at $x \in X$ if and only if whenever a filter F converges to x, the image filter $f(F)$ converges to $f(x)$ in y.
22. (a) Explain one-point compactification with an example.
    (b) Prove that an open subspace of a locally compact regular space is locally compact.

(3 × 5 = 15)