F 5533 (Pages : 3) Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2016

First Semester

Faculty of Science

Branch I (a)–Mathematics

MT 01 C 01—LINEAR ALGEBRA

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has 1 *weight.*

1. Write down the criteria for a subset of a vector space to be a subspace. If $V = \{(x, y, z) / x, y, z \in R\}$ and w is the set of all triplets such that $x - 3y + 4z = 0$, show that w is a subspace of V (R).
2. In a vector space show that $\alpha(v - w) = \alpha v - \alpha w$ and $1.0 = 0$.
3. Differentiate between linear transformation and linear fractional with examples.
4. Find the linear transformation $T : R^2 \to R^2$ given by T (2, 3) = (4, 5) and T (1, 0) = (0, 0).
5. Define even and odd permutations with example.
6. Explain the determinant of a linear transformation. If the determinant 'A' is invertible then for all B, $\det(A B A^{-1}) = \det B$. Prove.
7. Define invariant subspace with an example. Also state a necessary condition for a subspace to be invariant.
8. If $T^2 = T$ show that T is diagonalizable.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has 2 *weights.*

9. Suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of a linear transformation T in the basis (1, 0, 0) (0, 1, 0) (0, 0, 1) find the matrix of T in the basis (1, 1, 1) (0, 1, 1) (0, 0, 1).

Turn over

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10. Prove that the set $S = \{\alpha + i\beta, \gamma + i\delta\}$ is a basis for C (R) if and only if $\alpha\delta - \beta\gamma = 0$.

11. Show that the vectors $\alpha_1 = (1,0,-1)$; $\alpha_2 = (1,2,1)$; $\alpha_3 = (0,-3,2)$ form a basis for R^3. Express each of the standard basis vectors e_1, e_2, e_3 as a linear combination of $\alpha_1, \alpha_2, \alpha_3$ where $e_1 = (1, 0, 0)$; $e_2 = (0, 1, 0)$; $e_3 = (0, 0, 1)$.

12. Define the transpose of a linear transformation and show that it is linear as well as unique.

13. If A is invertible then for all B prove that $\det\left(A\,B\,A^{-1}\right) = \det B$ and $\det\left(A^{-1}\right) = (\det A)^{-1}$.

14. If a vector space is the direct sum of two of its subspaces. Obtain the basis for the vector space in terms of its component subspaces.

15. Let T be a linear operator on an n-dimensional vector space V. Prove that the characteristic and minimal polynomial for T have the same roots except for multiplicities.

16. Let V be two-dimensional over the field F of real numbers with a basis v_1, v_2. Find the characteristic roots and corresponding characteristic vectors for T defined by :

$Tv_1 = v_1 + v_2, Tv_2 = v_1 - v_2$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has 5 weights.

17. Let w be the subspace of C spanned by $\alpha_1 = (1,0,i)$ and $\alpha_2 = (1+i,1,-1)$.

    (a) Show that $\{\alpha_4\ \alpha_2\}$ is a basis for w.

    (b) Show that $\beta_1 = (1,1,0), \beta_2 = (1,i,1+i)$ is also a basis for w.

    (c) Find the co-ordinate of α_1 and α_2 in the ordered pair $\{\beta_1,\beta_2\}$ for w.

18. (a) Define algebra and describe the algebra of linear transformation. Also verify the axioms of algebra.

    (b) Let $T : V_3(R) \rightarrow V_3(R)$ defined by : T $(a, b, c) = (3a, a - b, 2a + b + c)$. Prove that T is invertible and find T^{-1}. Also prove $(T^2 - I)\ (T - 3\ I) = 0$.

19. Establish the isomorphism of a vector space and its double dual.

20. If D is any alternating n-linear function on $K^{n \times n}$, prove that for each $n \times n$ matrix A, $D(A) = (\det A)\, D(I)$, where I denotes the $n \times n$ identity matrix.

21. Characterise a diagonalizable linear operator on a finite dimensional vector space.

22. Explain elementary Canonical forms and obtain one of them.

$(3 \times 5 = 15)$