**QP CODE: 22000700**

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**Reg No** : ....................

**Name** : ....................

# M Sc DEGREE (CSS) EXAMINATION, APRIL 2022

## Third Semester

Faculty of Science

## CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

EBC92DD5

Time: 3 Hours | Weightage: 30

### Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Find the integral curves of* $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$
2. *Define orthogonal trajectories on the surface of a given system of curves.*
3. What is a Lagrange's equation? Why it is called so?
4. What do you mean by complete integral of a nonlinear partial differential equation? How will you derive the general integral and singular integral from a complete integral?
5. Find a complete solution of the equation $zpq = p + q$.
6. If $z = f(x + ay) + g(x - ay)$, show that $t = a^2 r$.
7. Solve $r + s - 2t - p - 2q = 0$.
8. Find the particular integral of $[D^2 - D']z = e^{2x+y}$.
9. Establish a formula for finding the potential function of a family of equipotential surfaces.
10. Prove that $r^{-2} cos\theta$ satisfy the Laplace's equation , when $r, \theta, \phi$ are spherical polar coordinates

(8×1=8 weightage)

### Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Verify that the equation* $a^2y^2z^2\ dx + b^2z^2x^2\ dy + c^2x^2y^2\ dz = 0$ *is integrable and if so find its primitive.*

12. Eliminate the arbitrary function $f$ from the given equations.
a) $f(x^2+y^2+z^2,\, z^2-2xy)=0$
b) $z=f(x-y)$

13. Show that the equations $xp=yq,\ z(xp+yq)=2xy$ are compatible and solve them.

14. Find the complete integral of the equation $xp+3yq=2(z-x^2q^2)$.

15. If $\beta_r D'+\gamma_r$ is a factor of $F(D,D')$ and $\phi_r(\varepsilon)$ is an arbitrary function of the single variable $\varepsilon$, then if $\beta_r \neq 0$, $u_r = exp\left(\frac{-\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$.

16. Reduce the equation to canonical form $u_{xx}=x^2u_{yy}$.

17. Prove that if $f(x,y,z)=c$ is a family of equipotential surfaces, then $\frac{\nabla^2 f}{|\nabla f|^2}$ is a function of $f$ alone.

18. State and prove the theorem of vanishing flux.

(6×2=12 weightage)

## Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove the following.
a) If $X$ is a vector such that $X\cdot curl X=0$ and $\mu$ is an arbitrary function of $x,y,z,$ then $(\mu X)\cdot curl(\mu X)=0$.
*b) A necessary and sufficient condition that the Pfaffian differential equation $X\cdot r=0$ should be integrable is that* $X\cdot curl X=0$.

20. Find the general solution of the differential equation $x(z+2a)p+(xz+2yz+2ay)q=z(z+a)$. Find also the integral surfaces which pass through the curves:
(a) $y=0, z^2=4ax$
(b) $y=0, z^3+x(z+a)^2=0$.

21. Solve by Jacobi's method, $z^2+zu_z-u_x^2-u_y^2=0$.

22. By separating the variables solve the one- dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2}=\frac{1}{k}\frac{\partial z}{\partial t}$

(2×5=10 weightage)