

QP CODE: 24018745



24018745

Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION , APRIL 2024

Second Semester

CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 Admission Onwards

6B32491C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. Differentiate between point wise convergence and uniform convergence of sequence of functions.
2. Find the symmetric points of i and 1 w.r.t the circle $|z - 3| = 1$.
3. Prove fundamental inequality for an arbitrary complex function $f(z)$.
4. If the closed curve γ lies inside of a circle, then prove that the index $n(\gamma, a)$ is zero for all points outside the circle.
5. Evaluate $\int_{|z|=2} \frac{dz}{z^2+1}$.
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int_{|z|=2} \frac{e^{2z}}{(z+1)^4} dz$.
7. Suppose that $f(z)$ is analytic in a region Ω' obtained by omitting a point a from Ω . State a necessary and sufficient condition for $f(z)$ to have an analytic extension in Ω .
8. Prove that the poles of a meromorphic function cannot have a limit point.
9. What do you mean by 'a cycle in a region is homologous to zero'?
10. Define connectivity of a multiply connected region.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.

11. Find the correspondence between the coordinates of a point on the Riemann sphere and its image in the complex plane.





12. Show that an analytic function $f(z)$ in a region W reduces to a constant if its modulus or argument is a constant.
 13. Define rectifiable arcs. State and prove the necessary and sufficient condition for an arc to be rectifiable.
 14. If $f(z)$ is an analytic function in an open disk Δ , then evaluate $\int_{\gamma} f(z) dz$ where γ is any closed curve in Δ .
 15. Show that an isolated singularity of $f(z)$ cannot be a pole of $e^{f(z)}$.
 16. State and prove the theorem on local correspondence.
 17. Write a brief note on calculus of residues.
 18. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?
- (6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. (i) Prove that the set of all linear transformations form a group under composition mapping.
 (ii) Show that a linear transformation $w = \frac{az+b}{cz+d}$, $c \neq 0$ is composed by a translation, an inversion, a rotation and a homothetic transformation.
 (iii) Show that cross ratio is invariant under a linear transformation.
 20. State and prove Cauchy's stronger theorem for rectangles.
 21. (a) State and prove the maximum principle.
 (b) Let $f(z)$ be analytic in a region Ω and $a \in \Omega$ such that $|f(a)| \leq |f(z)|$ for every $z \in \Omega$ then prove that either $f(a) = 0$ or $f(z)$ is a constant.
 22. Evaluate $\int_0^{\infty} \frac{x^{2m} dx}{x^{2n} + 1}$; $n > m$.
- (2×5=10 weightage)

