F 5534 (Pages : 2) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2016

First Semester

Faculty of Science

Branch : I (A) Mathematics

MT 01 C02—BASIC TOPOLOGY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five**.

Each question has weight 1.

1. Give examples : A set that is closed in the subspace may or may not be closed in the larger space.
2. Give two topologies on a topological space which are not comparable. Justify your answer.
3. Define second countable space. Show that every second countable space is separable.
4. Find a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous at precisely one point.
5. Prove or disprove : The interior and boundary of a connected set are connected.
6. Prove that the union of connected subspaces is also connected if they have a common point.
7. Compare Hausdorff property and separation axioms.
8. Prove that a topological space X is T_2 if and only if every singleton set $\{x\}$ is closed in X.

$(5 \times 1 = 5)$

Part B

Answer any **five**.

Each question has weight 2.

9. Show that finite intersection of open sets is open and arbitrary intersection of open sets need not be open.
10. Explain : Axiomatic characterisation of closure operators. Also give an example of dense set.
11. Let $f : X \to Y$ be a function from one topological space to another. Establish three equivalent conditions for f to be a homeomorphism.
12. Obtain the relationship between a compact subset and a compact space.

Turn over

13. Prove that the complement of $Q \times Q$ in the plane R^2 is connected. Also explain a chain connected metric space.

14. Define locally connected space and quotient space. Also show that every quotient space of a locally connected space is locally connected.

15. Explain the hierarchy of separation axioms.

16. Define Hausdorff space with two examples. Also prove that compact subsets in Hausdorff space are closed.

$(5 \times 2 = 10)$

Part C

Answer any **three**.

Each question has weight 5.

17. (a) For a subset A of a space X prove $\bar{A} = A \cup A'$.

 (b) Characterise open sets (i.e., those which are both closed and open) in terms of boundaries.

 (c) Characterise an open set of a topological space.

18. (a) Prove that every closed surjective map is a quotient map.

 (b) Every continuous real-valued function on a compact space is bounded and attains its extrema. Prove.

 (c) Explain : standard compactness argument.

19. (a) Prove that every continuous image of a compact space is compact.

 (b) Show that compactness property is weakly hereditary.

 (c) If $f : X \rightarrow Y$ is continuous, show that its graph is homeomorphic to X.

20. (a) Let $f : X \rightarrow Y$ be a continuous surjection prove, if X is connected then so is Y.

 (b) Show that [0, 1] is compact.

21. (a) Give example of a space which is regular but not completely regular. Prove your result.

 (b) Prove that every compact Hausdorff space is normal.

22. Prove that every regular Lindeloff space is normal.

$(5 \times 3 = 15)$