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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JULY 2017

Second Semester

Faculty of Science

Branch I (A) Mathematics

MT 02 C10—REAL ANALYSIS

(2012 Admission onwards)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Give example : Boundedness of f^1 is not necessary for f to be of bounded variation.
2. If f is monotonic in $[a, b]$, show that the set of discontinuities of f is countable.
3. If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, prove that $f \notin R$ on $[a, b]$ for $a < b$.
4. Define the Riemann-Stieltjes integral.
5. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
6. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval.
7. Evaluate $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$.
8. For $n = 0, 1, 2, \ldots$ and x real prove $|\sin nx| \leq n |\sin x|$.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Construct a continuous function which is not of bounded variation.
10. Establish the additive property of Riemann-Stieltjes integrals.

Turn over

11. Characterise all rectifiable curves.

12. Show that the integrability of f^2 does not imply the integrability of f but the integrability of f^3 imply the integrability of f.

13. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on the set E prove $\{f_n + g_n\}$ converges uniformly on E. In addition, if $\{f_n\}$ and $\{g_n\}$ are sequence of bounded function then prove $\{f_n g_n\}$ converges uniformly on E.

14. If $\{f_n\}$ is a sequence of continuous functions on E and $f_n \to f$ uniformly on E, prove that f is continuous on E.

15. Suppose $f(x)f(y) = f(x+y)$ for all real x and y. Assuming that f is differentiable and not zero prove $f(x) = e^{cx}$ where c is a constant.

16. Suppose $0 < \delta < \pi$, $f(x) = 1$ if $|x| \le \delta$, $f(x) = 0$ if $\delta < |x| \le \pi$ and $f(x + 2\pi) = f(x)$ for all x. Compute the Fourier Co-efficients of f.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Stating the three theorems to be used show that f is a function of bounded variation on $[a, b]$ if and only if f can be expressed as the difference of *two* increasing functions.

18. Obtain sufficient conditions for existence of Riemann-Stieltjes integrals.

19. Suppose $f \ge 0$, f is continuous on $[a, b]$ and $\int_a^b f(x)\,dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

20. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$. (a) For what values of x does the series converges absolutely ; (b) On what intervals does it converge uniformly ; (c) On what intervals does it fail to converge uniformly ; (d) Is f continuous wherever the series converges ; (e) Is f bounded ?

21. Establish the existence of a real valued continuous function which is nowhere differentiable.

22. Obtain the periods of the functions E, C and S. Also show that $E(Z_1)\,E(Z_2) = E(Z_1 + Z_2)$.

(3 × 5 = 15)