QP CODE: 22001450

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2022

First Semester

CORE - ME010103 - BASIC TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

D04AFB69

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a discrete metric on a non empty set X and find all open balls with respect to the discrete metric
2. Show that no sequence in scattered line topology on R can converges to an irrational number unless it is eventually constant
3. *Let B_1 and B_2 be two bases of two topologies τ_1 and τ_2 on X .Show that τ_1 is weaker than τ_2 if and only if every members of B_1 can be expressed as the union of some members of B_2*
4. Define derived set and accumulation point of a subset A of a topological space X.
5. Define (i) homeomorphism (ii) embedding.
6. Define weak topology.
7. Prove or disprove : Every Lindloff space is compact.
8. State Lebesgue covering Lemma
9. Show that every open subset of real line with usual topology can be expressed as the mutually disjoint union of open intervals
10. *Show that a space X is T_1 if and only if every singleton in X is closed*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define a discrete topological space.Show that a space is discrete if and only if every singleton is open
12. Characterize sub base of topological space
13. Prove that a second countable space always contains a countable dense subset.
14. Let the function $f : X \to Y$ is $\mathfrak{I} - \mathcal{U}$ continuous, where $\mathfrak{I}, \mathcal{U}$ be topologies on X, Y respectively. Then show that for all $V \in \mathcal{U},\ f^{-1}(V) \in \mathfrak{I}$ *if and only if* for any closed subset A of Y, $f^{-1}(A)$ is closed in X.
15. Prove that every second countable space is first countable
16. Given that a topological space X cannot be written as disjoint union of two non - empty opensets. Prove that X is connected.
17. Show that every path connected space is connected .What about the converse. Give justification
18. *Show that every T_3 spaces are T_2 but not conversely*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Show that every subspace of a metrisable space is metrisable
20. *(i) Define quotient map, quotient topology and quotient space.*
    (ii) Prove that every open surjective map is a quotient map.
    (iii) Prove that every closed surjective map is a quotient map.
21. (a) Prove that closure of a connected subset is connected.
    (b) Prove the following (1) Components are closed sets (2) Any two distinct components are mutually disjoint (3) Every nonempty connected set is contained in a unique component (4) Every space is the disjoint union of its components.
22. *State and prove hierarchy theorem of separation axioms*

(2×5=10 weightage)