QP CODE: 22001448

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2022

First Semester

CORE - ME010101 - ABSTRACT ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

8F914B39

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Prove that a direct product of abelian groups is abelian.*
2. *State Fundamental Theorem of Finitely generated abelian groups.*
3. *Define action of a group* G *on a set* X *and* G *– set.*
4. Let K and L be normal subgroups of G with $K \vee L = G$ and $K \cap L = \{e\}$. Show that $G/K \simeq L$ and $G/L \simeq K$.
5. Prove that every group of order 15 is cyclic.
6. Prove that no group of order 16 is simple.
7. *Use Fermat's theorem find the remainder of* 3^{47} *when it is divided by 23*
8. *Find all zeros of* $x^3 + 2x + 2$ *in* $\mathbb{Z}_7$
9. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and let C be the subring of F consisting of all the constant functions in F. Is C an ideal in F? Why?
10. Is $\mathbb{Q}[x]/(x^2 - 6x + 6)$ a field? Why?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Show that the set of all* $g \in G$ *such that* $i_g : G \to G$ *is the identity inner automorphism* i_e *is a normal subgroup of a group* G.

Turn Over

12. *Let* G *be a finite group and* X *a finite* G *– set. If* r *is the number of orbits in* X *under* G, *prove that* $r \cdot |G| = \sum_{g \in G} |X_g|$

13. Let G be a group of order p^n and let X be a finite G-set. Prove that $|X| \equiv |X_G| \pmod{p}$.

14. Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Prove that G is isomorphic to H × K.

15. *Show that for [(a,b)] and[(c,d)] in F the equations [(a,b)] +[(c,d)] = [(ad+bc,bd)] and [(a,b)] [(c,d)] = [(ac,bd)] give well defined addition and multiplication on F where F be the desired field of quotients of an integral domain D.*

16. *Is* $x^3 + 2x + 3$ *irreducible over* $\mathbb{Z}_5$*?. Why? Express it as a product of irreducible polynomials of* $\mathbb{Z}_5[x]$.

17. Show that if R, R' and R'' are rings and if $\phi : R \to R'$ and $\psi : R' \to R''$ are homomorphisms, then the composite function $\psi : R' \to R''$ is a homomorphism.

18. Prove that if R is a ring with unity and N is an ideal of R containing a unit, then N = R.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Let X be a G – set. Prove that $\{g \in G / gx = x\}$ is a subgroup of G for each $x \in X$.
(b) Let X *be a* G *– set and let* $x \in X$. *Prove that* $|Gx| = (G : G_x)$. *Also prove that if* $|G|$ *is finite, then* $|Gx|$ *is a divisor of* $|G|$.

20. (a) State and prove first Sylow theorem.
(b) Prove that no group of order p^r, for $r > 1$ is simple, where p is a prime.

21. *(a) State and prove the division algorithm in F[x].*
(b) Show that the multiplicative group of non zero elements of a finite field is cyclic.

22. If G is any group and R is a commutative ring with nonzero unity, then show that group ring (RG,+, .) is a ring.

(2×5=10 weightage)