QP CODE: 24018739

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , APRIL 2024

Second Semester

CORE - ME010201 - ADVANCED ABSTRACT ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

A7863F8E

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Check $\alpha = 1 + i$ is algebraic or transcendental over $\mathbb{R}$. If algebraic, find $deg(\alpha, \mathbb{R})$*
2. *Find the primitive 10^{th} roots of unity and primitive 5^{th} roots of unity in $\mathbb{Z}_{11}$*
3. State Ascending Chain Condition for a PID.
4. Show with an example that not every UFD is a PID.
5. Define a Euclidean domain. Give an example.
6. *Find all conjugates in $\mathbb{C}$ of $\sqrt{1+\sqrt{2}}$ over $\mathbb{Q}$.*
7. *Let $E = \mathbb{Q}(\sqrt{2})$ and $\sigma : E \to E$ defined by $\sigma(a + b\sqrt{2}) = a - b\sqrt{2}$ for $a, b \in \mathbb{Q}$. Find the fixed field of σ .*
8. *Define splitting field of a set of polynomials over a field F. Give an example.*
9. *Define group of a polynomial over a field.*
10. *Define symmetric function over a field F.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Define algebraic and finite extension of a field. Prove that a finite extension of a field F is an algebraic extension.*
12. *State and prove fundamental theorem of Algebra.*
13. Applying Euclidean algorithm find the gcd of 22,471 and 3,266.

14. Factor the given Gaussian integers into a product of irreducibles in $\mathbb{Z}[i]$

(a) 5 (b) 4 + 3i

15. *Describe all extensions of the automorphism* $\psi_{\sqrt{3},-\sqrt{3}}$ *of* $\mathbb{Q}(\sqrt{3})$ *to an isomorphism mapping* $\mathbb{Q}(i,\sqrt{3},\sqrt[3]{2})$ *onto a subfield of* $\overline{\mathbb{Q}}$ *.*

16. *If* $E \leq \overline{F}$ *is a splitting field over a field* F*, prove that every isomorphic mapping of* E *onto a subfield of* $\overline{F}$ *and leaving* F *fixed is actually an automorphism of* E*. Further prove that if* E *is a splitting field of finite degree over* F*, then* $\{E:F\} = |G(E/F)|$.

17. If E is a finite extension of a field F, then prove that $\{E:F\}$ divides $[E:F]$.

18. *Let* E *be a finite separable extension of a field* F*. Prove that there exists an* $\alpha \in E$ *such that* $E = F(\alpha)$*.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. *Prove that the field* F *of constructible real numbers consists precisely of all real numbers that we can obtain from* $\mathbb{Q}$ *by taking square roots of positive numbers a finite number of times and applying a finite number of field operations.*

20. a) Prove that every PID is a UFD.

b) If D is a UFD, then prove that for every nonconstant f(x) in D[x], f(x) = (c)g(x), where c belongs to D and g(x) in D[x] is primitive. Also prove that the element c is unique upto a unit factor in D and g(x) is unique upto a unit factor in D.

21. a) State and prove the isomorphism extension theorem.

b) Let $\overline{F}$ *and* $\overline{F}'$ *be two algebraic closures of a field* F*. Prove that* $\overline{F}$ *is isomorphic to* $\overline{F}'$ *under an isomorphism leaving each element of* F *fixed.*

22. Prove the following.

a) Every field of characteristic zero is perfect.

b) Every finite field is perfect.

(2×5=10 weightage)