G 17003087

17003087

Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JULY 2017

Second Semester

Faculty of Science

Branch I (A) Mathematics

MT02 C06—ABSTRACT ALGEBRA

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight **1**.

1. Examine whether $Z_{12} \times Z_5$ is isomorphic to Z_{60}.
2. State the fundamental theorem of finitely generated abelian groups.
3. Define extension field with *two* examples.
4. Establish : Squaring the circle is impossible.
5. Define Sylow p-subgroup.
6. Prove : The center of a non-trivial p-group G is non-trivial.
7. Define splitting field with an example.
8. Define Klein 4-group and Galois group of K over **F**.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that $Z_2 \times Z_2$ is not cyclic but isomorphic to the Klein 4-group.
10. Evaluate $(x+1)^2$ and $(x+1)+(x+1)$ in $\mathbb{Z}_2[x]$.
11. State and prove the theorem to show how any finite field can be formed from the prime subfield.
12. Prove : Trisecting the angle is impossible.
13. State isomorphism extension theorem use it to show that an algebraic closure of **F** is unique.
14. When two elements of an algebraic extension of a field are conjugate ? Explain with conjugate complex numbers ?

Turn over

15. Prove : If E is a finite extension of F then E is separable over F if and only if each α in E is separable over F.

16. Show that if $[E:F] = 2$ then E is a splitting field over F.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Characterise reducible polynomial of degree 2 or 3 in $F[x]$.

    (b) Characterise a maximal ideal $\langle p(x) \rangle \neq \{0\}$ in $F(x)$.

18. Establish necessary and sufficient conditions for a group to be the internal direct product of two subgroups.

19. State and prove Kronecker's theorem on extension field. Illustrate the construction involved with two examples.

20. Establish the existence of a finite field of order p^r for every prime power p^r, $r > 0$. Prove the lemma used.

21. State and prove the theorem on the basic isomorphisms of algebraic field theory.

22. State and prove Primitive Element Theorem. Deduce that a finite extension of a field of characteristic zero is a simple extension.

$(3 \times 5 = 15)$