



23003109

QP CODE: 23003109

Reg No : .....

Name : .....

**M Sc DEGREE (CSS) EXAMINATION, APRIL 2023****First Semester****CORE - ME010101 - ABSTRACT ALGEBRA**

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

5053C2A9

Time: 3 Hours

Weightage: 30

**Part A (Short Answer Questions)***Answer any **eight** questions.**Weight 1 each.*

1. Find all abelian groups upto isomorphism of order 360.
2. Let  $X$  be a  $G$ -set. Define an orbit in  $X$  under  $G$ .
3. Define torsion coefficients of a finite abelian group. Find the torsion coefficients of  $\mathbb{Z}_6 \times \mathbb{Z}_{12} \times \mathbb{Z}_{20}$
4. Find the kernel of the homomorphism  $\phi: \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12}$ , where  $\phi(1) = 10$ .
5. Prove that every group of prime-power order is solvable.
6. Prove that no group of order 20 is simple
7. Compute the evaluation homomorphism  $\phi_2(x^2 + 3)$ ,  $F = E = \mathbb{C}$
8. Find the sum and product of the polynomials  $f(x) = 2x^2 + 3x + 4$  and  $g(x) = 3x^2 + 2x + 3$  in  $\mathbb{Z}_6[x]$
9. Define ring of endomorphisms.
10. Is  $\mathbb{Q}[x]/(x^2 - 5x + 6)$  a field? Why?

(8×1=8 weightage)





### Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Let  $H$  be a normal subgroup of a group  $G$ . Prove that the cosets of  $H$  form a group  $G/H$  under the binary operation  $(aH)(bH) = (ab)H$ .
12. Let  $X$  be a  $G$ -set. When we say that  $G$  acts faithfully on  $X$ ? Show that  $G$  acts faithfully on  $X$  if and only if no two distinct elements of  $G$  have the same action on each element of  $X$ .
13. Prove that any two Sylow  $p$ -subgroups of a finite group are conjugate.
14. If  $H$  and  $K$  are finite subgroups of a group  $G$ , then prove that  $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$ .
15. Let  $D$  be a given integral domain and let  $S$  be the subset of  $D \times D$  given by  $S = \{(a,b)/a,b \in D, b \neq 0\}$ . Two elements  $(a,b) \sim (c,d)$  in  $S$  if and only if  $ad = bc$ . Show that  $\sim$  is an equivalence relation.
16. State and prove the Eisenstein criterion for irreducibility.
17. Show that  $\mathbb{Z}_2 G$  is a group algebra where  $G$  is the group  $\{e, a\}$ .
18. Prove that a field contains no proper non trivial ideals.

(6×2=12 weightage)

### Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. (a) Prove that a direct product of a finite number of abelian groups forms an abelian group.  
(b) Prove that the group  $\mathbb{Z}_m \times \mathbb{Z}_n$  is cyclic and isomorphic to  $\mathbb{Z}_{mn}$  if and only if  $\gcd$  of  $m$  and  $n$  is 1.
20. (a) State and prove Cauchy's theorem.  
(b) Let  $G$  be a finite group. Prove that  $G$  is a  $p$ -group if and only if  $|G|$  is a power of  $p$ .
21. State and prove Fermat's little theorem. Find the remainder of  $7^{1000}$  when divided by 24





22. (a) Let  $\phi : R \rightarrow R'$  be a ring homomorphism with kernel  $H$ . Then show that the additive cosets of  $H$  form a ring  $R/H$ . Also prove

that the map  $\mu : R/H \rightarrow \phi[R]$  defined by  $\mu(a + H) = \phi(a)$  is an isomorphism.

(b) Prove that for a subring  $H$  of  $R$ , the multiplication of additive cosets of  $H$  is well defined by the equation  $(a + H)(b + H) = ab + H$

if and only if  $ah \in H$  and  $hb \in H$  for all  $a, b \in R$  and  $h \in H$ .

(2×5=10 weightage)

