QP CODE: 23145320

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, DECEMBER 2023

First Semester

CORE - ME010102 - LINEAR ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

D4B6BF1D

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let V be the set of pairs (x, y) of real numbers and let F be the field of real numbers. Define (x, y) + (x_1, y_1) = (x + x_1, 0) and *c(x, y) = (cx, 0). Is V, with these operations, a vector space over F?*
2. *Define dimension of a vector space. What is the dimension of the space of all n x n matrices over F.*
3. Find the range, rank,null space and nullity of the identity operator in a finite dimensional vector space V.
4. Let V and W be finite dimensional vector spaces over the field F .If V and W are isomorphic, prove that $dimV = dimW$
5. If $T : R^2 \to R^2$ is a linear operator defined as $T(x_1, x_2) = (-x_2, x_1)$,find the matrix of T in the ordered basis $\{(1, 2), (1 - 1)\}$for R^2
6. Check whether the function D on $\mathbb{R}^{3\times 3}$ defined by *D(A) =* $A_{11} A_{22} A_{33}$ *is 3-linear.*
7. State the theorem which describes the construction of a determinant function on K^{nxn}, if given such a function on $K^{(n-1) \times (n-1)}$, *where K is a commutative ring with identity.*
8. *Prove that the determinant of the Vandermonde matrix* $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ *is (b - a)(c - a) (c - b).*

9. *Find the characteristic values, if any, of the linear operator* T *on* $\mathbb{R}^2$ *which is represented in the standard ordered basis by the matrix* $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

10. *Find a projection* E *which projects* $\mathbb{R}^2$ *onto the subspace spanned by* $(1, 1)$ *along the subspace spanned by* $(1, -2)$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let W be the subspace of $\mathbb{C}^3$ spanned by $\alpha_1 = (1, 0, i)$ and $\alpha_2 = (1 + i, 1, -1)$.
    (a) Show that α_1 and α_2 form a basis for W.
    (b) Show that the vectors $\beta_1 = (1, 1, 0)$ and $\beta_2 = (1, i, 1 + i)$ are in W and form another basis for W.
    (c) What are the coordinates of α_1 *and* α_2 *in the ordered basis* $\{\beta_1, \beta_2\}$ *for W?*

12. *Show that the non-zero row vectors of a non-zero row-reduced echelon matrix R form a basis for the row space of R.*

13. Prove that a linear transformation $T : V \to W$,where V and W are vector spaces over the same field, is non-singular if and only if it is one-one

14. Let f and g be linear functionals on a vector space V. If the null space of g contains the null space of f, prove that g is a scalar multiple of f.

15. Let V and W be finite dimensional vector spaces over the field F and $T : V \to W$ is a linear transformation. Prove that $Rank(T^t) = Rank\ T$.

16. *If K is a commutative ring with identity, show that the determinant of an n x n matrix over K and its transpose are the same.*

17. *Let* T *be a linear operator on* $\mathbb{R}^3$ *which is represented in the standard ordered basis by the matrix* $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ *. Prove that* T *is diagonalizable by exhibiting a basis for* $\mathbb{R}^3$ *, each vector of which is a characteristic vector of* T

18. *Find the Characteristic and Minimal polynomials of the matrix* $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. (a) State and prove a necessary and sufficient condition for the union of two subspaces of a vector space V to be a subspace of V.

(b) Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let V_e be the subset of even functions and let V_o be the subset of odd functions.

(i) Prove that V_e and V_o are subspaces of V.

(ii) Prove that $V_e + V_o = V$.

(iii) Prove that $V_e \cap V_o = \{0\}$.

20. Let V be a finite dimensional vector space over the field F, then prove that

(i) $dimV = dimV^*$ by exhibiting a unique dual basis $B^* = \{f_1, f_2, f_3 \ldots f_n\}$ for any ordered basis $B = \{\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n\}$ of V and

(ii) $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and $\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i$ where *f* is any linear functional and α is any vector in V.

21. *If D is any alternating n-linear function on* $K^{n \times n}$*, then prove that for each n x n matrix A, D(A) =(det A)D(I).*

22. *Characterize a diagonalizable linear operator on a finite dimensional vector space.*

(2×5=10 weightage)