



**QP CODE: 22002353**

**Reg No** : .....

**Name** : .....

**MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2022**

**Second Semester**

**CORE - PH010203 - STATISTICAL MECHANICS**

M Sc PHYSICS, M.Sc. SPACE SCIENCE

2019 Admission Onwards

64C49064

Time: 3 Hours

Weightage: 30

**Part A (Short Answer Questions)**

*Answer any **eight** questions.*

*Weight **1** each.*

1. Calculate the chemical potential from the Sakur-Tetrode formula for entropy and show that it is an intensive quantity.
2. Write down the ensemble average  $\langle f \rangle$  of a physical quantity  $f(q, p)$ , in phase space.
3. Write down the canonical partition function for a system with continuous energy distribution.
4. Discuss the variation of relative fluctuation in energy for a system in canonical ensemble as a function of the number of particles  $N$ .
5. Write down the probability distribution for grand canonical ensemble.
6. Show thermodynamically that  $SdT - VdP + Nd\mu = 0$ .
7. Explain why the Gibbs correction in quantum statistics is inadequate to account for the indistinguishability.
8. Explain the Debye's modification of Einstein's model for specific heat capacity of solids.
9. Show that the Fermi momentum  $p_F$  is proportional to  $n^{\frac{1}{3}}$ .
10. For a beam of electrons with initial kinetic energy  $E$  impinging on a metal of barrier height  $W$ , show that the refractive index  $n = \left( \frac{E+W}{E} \right)^{1/2}$ .

(8×1=8 weightage)





### Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Consider a system of one dimensional harmonic oscillators with frequency  $\omega$  in microcanonical ensemble. Calculate the entropy  $S(E, V, N)$  and pressure  $P$  of the system.
12. A system has four non-degenerate energy levels. The energy levels are  $E_1 = 0$ ,  $E_2 = 1.4 \times 10^{-23} J$ ,  $E_3 = 4.2 \times 10^{-23} J$  and  $E_4 = 8.4 \times 10^{-23} J$ . Given that the system is at a temperature of  $5K$ , what is the probability that the system is in the  $E_1 = 0$  level?
13. Show the following  $\langle \sum_i q_i \dot{p}_i \rangle = -3NkT$  and  $\langle \sum_i p_i \dot{q}_i \rangle = 3NkT$ , where  $q_i$  and  $p_i$  are the generalised coordinates and momenta of a classical system.
14. Show that  $i\hbar \dot{\hat{\rho}}(t) = [\hat{H}, \hat{\rho}]$ , where  $\hat{\rho}$  is the density matrix and  $\hat{H}$  is the Hamiltonian of the system.
15. Obtain the ensemble average of an operator  $G$  for a system in grand canonical ensemble in terms of the canonical ensemble average  $\langle G \rangle_N$ .
16. Show that the most probable number of particles per energy level in the  $i^{th}$  cell is given by  $\frac{n_i^*}{g_i} = \frac{1}{e^{(\alpha + \beta \epsilon_i)} + a}$ .
17. Discuss the nature of isotherms of an ideal Bose gas.
18. Show that the energy density of a blackbody radiation from a cavity is proportional to the fourth power of temperature.

(6×2=12 weightage)

### Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Establish the link between statistics and thermodynamics through the total number of microstates  $\Omega$ . Derive the thermodynamic quantities using the definition  $S = k \ln \Omega$ .
20. Obtain various thermodynamic quantities for a system of non-interacting indistinguishable particles in grand canonical ensemble, if the canonical single particle partition function is  $Q_1(V, T) = V f(T)$ . Obtain these quantities if  $f(T) = cT^n$ , where  $c$  is a constant and  $n$  is a real number.
21. Obtain the general expression for paramagnetic susceptibility of ideal Fermi gas. Discuss the nature of susceptibility at low and high temperatures.
22. Discuss the salient features of first order phase transition. Deduce Clapeyron equation.

(2×5=10 weightage)

