F 5132

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY/FEBRUARY 2017

First Semester

Faculty of Science

Branch II–Physics–A–Pure Physics

PH 1C 01—MATHEMATICAL METHODS IN PHYSICS–I

(2012 Admission onwards)

Time : Three Hours   Maximum Weight : 30

Part A

Answer any **six** *questions.*
Weight 1 each.

1. Define Levi-Civita Tensor.

2. What is the physical interpretation of curl of a vector ?

3. If x^i and x^{-i} are independent co-ordinate of a point, show that $\dfrac{\partial x^j}{\partial x^{-k}} \dfrac{\partial x^{-k}}{\partial x^i} = \delta_i^j$.

4. Define Hilbert space.

5. Verify Green's theorem for $\displaystyle\int_C \left[\left(xy + y^2 \right) dx + x^2 \, dy \right]$ where C is bounded by $y = x$ and $y = x^2$.

6. Define mixed tensor. Show that kronecker delta is a mixed tensor of order two.

7. Write different forms of β function.

8. Write a note on Schwartz inequality.

9. Define random variable in probability distribution.

10. Define normal matrices.

$(6 \times 1 = 6)$

Part B

Answer any **four** *questions.*
Weight 2 each.

11. Find the matrix P which transforms the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ to the diagonal form.

Turn over

12. If $f = \left(x^2 + y^2 + z^2\right)^{-n}$, find div grad f and determine the value of n if div grad $f = 0$.

13. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

14. Prove that $H'_n(x) = 2nH_{n-1}(x)$.

15. Show that Legendre polynomial can be expressed as $P_n(X) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n} \left(x^2 - 1\right)^n$.

16. Obtain the expression for metric tensor in cylindrical co-ordinate system.

$(4 \times 2 = 8)$

Part C

Answer all questions.
Weight 4 each.

17. (A) Explain :

   (i) Addition of tensors.

   (ii) Outer product of tensors.

   (iii) Contraction of tensors.

   (iv) Inner product of tensors.

Or

   (B) (i) Define Christoffel's symbols. Explain Riemann-Christoffel symbol.

   (ii) Define geosedic. Obtain differential equation of geosedic in a space.

18. (A) (i) Prove that $\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\, dx = 2^n n! \sqrt{\pi}$.

   (ii) Obtain generating function of Legendre polynomial.

Or

   (B) (i) Prove that beta function is symmetric.

   (ii) Obtain the relation between beta and gamma function.

19. (A) Derive the solution of Bessel differential equation.

Or

(B) (i) What are unitary and orthogonal transformation ?

(ii) Explain normal modes of vibration.

20. (A) (i) Explain Poisson and Gaussian distribution.

(ii) Derive the theorem corresponding to relation between line and surface integral.

Or

(B) Explain cylindrical co-ordinate system. Prove that this co-ordinate system is orthogonal. Express $zi - 2xj + yk$ in cylindrical polar co-ordinate system.

$(4 \times 4 = 16)$