QP CODE: 24018799

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Reg No : .....

Name : .....

MSc DEGREE (CSS) EXAMINATION , APRIL 2024

Second Semester

CORE - PH010202 - QUANTUM MECHANICS-I

M Sc PHYSICS,M.Sc.SPACE SCIENCE

2019 Admission Onwards

C87BB581

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What is a projection operator? Comment on the eigenvalues of projection operator.
2. Write down the properties of Infinitesimal translation operator.
3. Prove that the wave functions in the position space and momentum space are Fourier transforms of each other.
4. Is time an observable in quantum mechanics? Explain.
5. Give the solution of the Schrodinger equation for the time evolution operator if the Hamiltonian of the system is time independent.
6. Consider, a spin half system subjected to a magnetic field in the z direction. At time $t = 0$ the state of the system is given by $|\alpha, t_0 = 0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$, where $|\pm\rangle$ are the S_z eigenstates. Calculate $\langle S_z \rangle$ system at $t > 0$.
7. Can we derive the time-energy uncertainty relation from the general uncertainty relation? Why?
8. Obtain the period for spin precession $\tau_{\text{presession}}$ for a spin $1/2$ particle placed in a magnetic field B, which is in the z-direction.
9. Show that for Pauli matrices $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$.
10. Write down the matrix representation of J_- in the $\{|j, m\rangle\}$ basis.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. A particle of mass m confined in an infinite potential well of length a has the wave function $\psi(x) = \langle x|\phi_n\rangle = \frac{A}{\sqrt{a}}\phi_1(x) + \sqrt{\frac{3}{5}}\,\phi_3(x) + \sqrt{\frac{1}{10}}\,\phi_5(x)$. The energy eigenfunctions $\phi_n = \langle x|\phi_n\rangle = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$ form an orthonormal basis.

    (i)Find A so that $\psi(x)$ is normalised.(ii) Find the possible results of measurement of energy and the corresponding probabilities. (iii) Find the expectation value of energy.

12. Prove that the product of two unitary operators is unitary.

13. Calculate the uncertainty product $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ corresponding to the position state wave function $\langle x'|\alpha\rangle = \frac{1}{\pi^{1/4}\sqrt{d}}e^{-\frac{x'^2}{2d^2}}$.

14. Let $x(t)$ be the coordinate operator for a free particle in Heisenberg picture. Evaluate $[x(t), x(0)]$.

15. Show that the uncertainty product for the energy eigen states of a harmonic oscillator $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle = (n+1/2)^2\hbar^2$.

16. Evaluate $\vec{J}\times\vec{J}$, where $\vec{J}$ is the angular momentum operator.

17. Show thatJ^2 and J_z can be simultaneously diagonalized.

18. Show that $\langle x'|(\mathbf{x}.\,\mathbf{p})^2|\alpha\rangle = -\hbar^2\left(r^2\frac{\partial^2}{\partial r^2}\langle x'|\alpha\rangle + r\frac{\partial}{\partial r}\langle x'|\alpha\rangle\right)$, where $\mathbf{x}$ and $\mathbf{p}$are the position and momentum operators.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Discuss the Analogy of the Sequential Stern-Gerlach experiment with the polarization of light.

20. Derive Ehrenfest's theorem using Heisenberg picture and then express it in a form which is valid for both Heisenberg and Schrodinger pictures. Discuss the significance of the theorem.

21. Discuss the general theory of angular momentum addition and develop the idea of the Clebsch-Gordon coefficients.Discuss the properties of the Clebsch-Gordon coefficients. Also derive the recursion relations.

22. Obtain the radial wave equation for a system moving under a central potential. Discuss the behaviour of the radial wave function near the origin .

(2×5=10 weightage)